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A four-equation friction model for water hammer calculation in quasi-rigid pipelines

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ABSTRACT

Friction coupling affects water hammer evolution in pipelines according to the initial flow regime. Unsteady friction models are only validated with uncoupled formulation. On the other hand, coupled models such as four-equation model, provide more accurate prediction of water hammer since fluid-structure interaction (FSI) is taken into account, but they are limited to steady-state friction formulation. This paper deals with the creation of the "four-equation friction model" which is based on the incorporation of the unsteady head loss given by an unsteady friction model into the four-equation model. For transient laminar flow cases, the Zielke model is considered. The proposed model is applied to a quasi-rigid pipe with axial moving valve, and then calculated by the method of characteristics (MOC). Damping and shape of the numerical solution are in good agreement with experimental data. Thus, the proposed model can be incorporated into a new computer code.

1. Introduction

Water hammer problem is relevant to various branches of industry such as water-supply-networks, industrial conduits, cooling circuits of thermal and nuclear power plants, etc. Perturbation in the fluid flow is produced due to several operations such as starting or failure in pump and turbine and also fast opening or closing of the valve. Mechanical loadings on pipe systems caused by water hammer belong to the most important and most difficult to calculate design. Severe water hammer often may cause a rupture of piping components, service pipe failures, joint failures and other damage to the hydraulic system. This is why water hammer must be continuously controlled and predicted. Surge suppressors, relief valves, slow closing gate and cone type valves are examples of water hammer prevention equipments.

In order to ensure the global economic efficiency and safety operations of hydraulic systems, several mathematical contributions are presented. A number of physical parameters can be taken into account. These include dissolved and free air in the liquid, unsteady friction, fluid-structure interaction (FSI), viscoelastic behavior of the pipe-wall material and cavitation. Water hammer problems are usually simulated using one-dimensional water hammer equations based on the quasi-steady friction model. The main assumption of this method is that the head loss during water hammer phenomenon is equal to the head loss obtained from the steady flow. However, this assumption is not valid for most of water hammer problems due to the existence of strong gradients and reverse flows near the pipe wall.

Unsteady friction models are the subject of various research projects in research centers all over the world. The most widely used models consider extra friction losses to depend on a history of weighted accelerations during unsteady phenomena, or on instantaneous flow acceleration. The first group of models was developed by Zielke [1]. These models consider the instantaneous wall shear stress, which is directly proportional to friction losses, is the sum of the quasi-steady value and a term in which certain weights are given to the past velocity changes [1]. This approach is assigned for transient laminar flow cases. The Zielke model is based on solid theoretical fundamentals and the multiple experimental validation tests have shown good conformity between calculated and measured results. As demonstrated [2], Adamkowsky and Lewandowsky validated the Zielke model and other several unsteady friction models against experimental results without junction coupling (fixed valve).

The objective of this work is to prove the insufficiency of both Zielke model and four-equation model to simulate water hammer problems with junction coupling (free moving valve) and to

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Nomenclature

Abbreviations

FSI	Fluid-Structure Interaction
MOC	Method of Characteristics
SLI	Space-line interpolation
TLI	Time-line interpolation
WSA	Wave-speed adjustment

Scalars

γ	Pipe inclination
ν	Poisson coefficient
σ	Stress
Σ	Real positive number
ρ	Mass density
λ	Characteristic direction
c	Anchor coefficient
A	Section
C	Celerity
D	Inner diameter of the pipe
E	Pipe thickness
E	Young's modulus
F	Friction coefficient of Darcy-Weisbach
g	Gravitational acceleration
h	Unsteady head loss
i	Space increment
j	Time increment
K	Fluid bulk modulos
L	Pipe length
M	Mass
p	Relative pressure in the fluid
R	Inner radius of the pipe
T	Time
u	Displacement
V	Cross-sectional averaged fluid velocity
W	Weighting function
Z	Axial co-ordinate

Matrices and vectors

$\mathbf{A}, \mathbf{B}$	Stiffness matrices of the hyperbolic linear system
$\mathbf{k}$	Second part of the vector **s**
$\mathbf{M}$	Stiffness matrix of the algebraic system
$\mathbf{n}$	First part of the vector **s**
$\mathbf{N}$	Matrix used to find the vector **n**
$\mathbf{r}$	Right hand side vector of the hyperbolic linear system
$\mathbf{s}$	Right hand side vector of the algebraic system
$\mathbf{y}$	Transformed vector of unknowns
$\mathbf{Y}$	Matrix used to express the matrix **N**

Subscripts

F	Fluid
$0, L$	Boundary position
P	Pipe wall
z	Axial direction

validate a proposed model called “Four-equation Friction Model”. The proposed model will be calculated in a quasi-rigid pipeline with axial vibration using the MOC, where water hammer is caused by an instantaneous valve closure. The computation results will be compared against experimental data.

2. Theory

2.1. Unsteady friction models

Unsteady friction derives from the extra losses caused by the two-dimensional nature of the unsteady velocity profile [1–3]. Many types of unsteady friction models exist in the literature. As demonstrated [1], Zielke has developed the convolution-based unsteady friction model which is based on analytical solutions obtained for laminar flow (Re $\le$ 2320). By assuming the velocity V is uniform on each section A, Zielke defined the head loss h_f as

$$h_f = \frac{f}{2gD} V|V| + \frac{16\nu}{gD^2}\left(\frac{\partial V}{\partial t} * W\right)(t) \quad (1)$$

where W is the weighting function in time and (*) denotes convolution.

As described [3], the friction head loss can be thought of as comprising a steady part and an unsteady part. The first part of Eq. (1) is the Darcy-Weisbach formulae defining the quasi-steady head loss per unit, denoted $h_{f\cdot q}$ in this issue. Whereas the second part is the unsteady head loss per unit, denoted $h_{f\cdot u}$. This term follows from the convolution of the weighting function W with past temporal velocity variations $\partial V/\partial t$. Adamkowsky and lewandowsky call it *pipeline inertance* [2].

To summarize, Eq. (1) can be written as

$$h_f = h_{f\cdot q} + h_{f\cdot u} \quad (2)$$

The development of the convolution relation gives [2,3]

$$\left(\frac{\partial V}{\partial t} * W\right)(t) = \int_0^t \frac{\partial V}{\partial t}(u)\cdot W(t-u)du \quad (3)$$

The weighting function is defined [1]

$$W(\tau) = \begin{cases} \sum_{j=1}^{6} m_j \tau^{j/2-1} & \text{for } \tau < 0,02 \\ \sum_{j=1}^{5} e^{-n_j\tau} & \text{for } \tau > 0,02 \end{cases} \quad (4)$$

in which $n_j = \{26{,}3744;\ 70{,}8493;\ 135{,}0198;\ 218{,}0198;\ 322{,}5544\}$, $\tau = 4\nu t/D^2$ and $m_j = \{0{,}282095;\ -1{,}25;\ 1{,}057855;\ 0{,}9375;\ -0{,}396696;\ -0{,}351563\}$

The convolution-based unsteady frictional head loss term h_f in a staggered characteristic grid, called *full convolution scheme* [3] is

$$h_f(z,t) = \frac{fV(z,t)|V(z,t)|}{2gD} + \frac{16\nu}{gD^2}\sum_{j=1,3,5,\ldots}^{M}[V(z,t-j\Delta t+\Delta t) - V(z,t-j\Delta t-\Delta t)]W_0(j\Delta t) \quad (5)$$

with $M = t/\Delta t - 1$.

It is worth noting that several friction models are studied in literature, such as Vardy and Brown model, Brunone model, Zarzycki model and others [2–4].