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Mathematical modeling for heat conduction in stone fruits

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ABSTRACT

Stone fruit is cooled after harvesting to extend its shelf life and prevent postharvest losses. Because it is quickly subject to chilling injuries at inappropriate temperatures, its thermal properties should be known in order to design an optimum cooling process. However, how long does it take for an olive to reach its storage temperature at the stone–pulp interface? This paper proposes approximated equations as a model for predicting cooling times at the stone–pulp interface and for measuring the thermal diffusivity of the pulp and the external heat transfer coefficient. The model is based on a solution in Fourier series for the conduction of heat in spheres with an inner concentric, insulating spherical core, as a model of conduction of heat in stone fruits.

Modélisation mathématique de la conduction thermique dans les fruits à noyau

Mots clés : Fruits à noyau ; Transfert de chaleur ; Séries de Fourier ; Paramètres thermophysiques ; Coefficient de transfert de chaleur en surface

1. Introduction

In postharvest processing of plant foods, rapid cooling is generally necessary to extend their shelf life and reduce losses (Erdoğdu et al., 2014). In the design of this processing there are basic control parameters that are necessary to predict accurately, such as the cooling time needed to reach a certain temperature at the thermal center of the product, the average temperature of the product at a given time, the amount of heat to be extracted to reach that temperature, etc. In most cases, the calculation of these parameters is based on the first approximation to the general Fourier series solution for regular geometries in homogeneous isotropic bodies, which usually does not explicitly include the stone. In addition, the indirect measurements of thermal diffusivity and the external heat transfer coefficient are

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Nomenclature

A_n	series expansion constants
$\bar{A}_n$	series expansion constants (mass average)
a	radius of the stone [m]
$Bi = h{\cdot}R/k$	Biot number
c	specific heat capacity [$\mathrm{Wkg^{-1}K^{-1}}$]
$Fo = \alpha \cdot t/R^2$	Fourier number
Fo_Y	Fourier number required to reach a dimensionless temperature Y
h	surface heat transfer coefficient [$\mathrm{Wm^{-2}K^{-1}}$]
k	thermal conductivity [$\mathrm{Wm^{-1}K^{-1}}$]
m	intermediate variable appearing in Eq. (39)
$M_n = [(n-1/2)\pi]^2$	constants appearing in Appendix C, Eq. (C-4) (dimensionless)
$Q(t)$	total energy extracted up to the moment t
$\dot{Q}$	heat flow transferred through the surface at the moment t
R	radius of the body [m]
r	radius from the center of the fruit [m]
S	surface [$\mathrm{m^2}$]
S	in Eq. (22): absolute slope of the linear portion in semi-logarithmic scale [$\mathrm{s^{-1}}$]
t	time [s]
T	temperature [°C]
T_{ex}	temperature of the medium [°C]
ΔT_0	$(T_0 - T_{ex})$
V	volume [$\mathrm{m^3}$]
$x = r/R$	dimensionless distance from the center
$x_a = a/R$	dimensionless distance of the stone–pulp contact surface
$Y = (T - T_{ex})/(T_0 - T_{ex})$	dimensionless ratio of temperature difference
$\bar{Y} = (\bar{T} - T_{ex})/(T_0 - T_{ex})$	dimensionless ratio of mass average temperature difference

Greek letters

$\alpha = k/(\rho \cdot c)$	thermal diffusivity [$\mathrm{m^2s^{-1}}$]
δ^2	(minus) slope of the linear portion on semi-logarithmic scale (dimensionless)
$\delta_n^{\,2}$	solutions to the transcendental equation of boundary condition (dimensionless)
$\delta_{n,Max} = \delta_n$ for $Bi \to \infty$	(dimensionless)
ρ	density [$\mathrm{kg\,m^{-3}}$]
$\psi = \psi(\delta_n x)$	spatial component of the solution in Fourier series expansion

Subscripts

0	value when $t = 0$
1	first term in infinite series
a	value on the stone–pulp contact surface
exp	experimental
Max	maximum value that can be achieved
min	minimum value that can be achieved
n	index in infinite series
sf	value at $r = R$
∞	for $Bi \to \infty$

also based on these approximations (Awuah et al., 1995; Erdoğdu, 2005, 2008). Therefore, it would be useful to have appropriate mathematical models that explicitly include the stone, to allow accurate prediction of the design parameters mentioned above, and to measure the thermal diffusivity and the film coefficient.

In fact, among the analytical solutions to these problems in solids with elementary geometries there is a considerable body of literature on approximation models. Since the first published results in the 1960s (Gac, 1963; Pflug et al., 1965; Smith et al., 1967), these models have been based on a linear approximation of the cooling kinetics (on a semi-logarithmic scale), valid from a given time onwards. The main applications of analytical approximations of this kind are: estimation of cooling/heating times, indirect measurement of the surface heat transfer coefficient h when the thermophysical parameters are known, and measurement of thermal parameters if h is known. In general, these approximation equations are valid for solids with elementary shapes (Becker and Fricke, 2002; Erdoğdu, 2005, 2008; Kondjoyan, 2006). These approximate equations were extended to ellipsoidal or even irregular geometries in some works (Cleland and Earle, 1982; Cuesta et al., 1990; Cuesta and Lamúa, 1995, 2002; Lin et al., 1996a, 1996b, 2000; Smith et al., 1967; Yilmaz, 1995 (and the *Letter to the Editor* – about this last paper – published by Van Beek and Meffert, 1997).

In stone fruits, the theoretical thermal problems are different from those of solid products. In fact, stone fruits, such as olives, cherries, plums, and so forth, have approximately spherical or ellipsoid geometries, but inside they contain a ligneous core – the seed – whose physical and thermophysical parameters are radically different from those of the edible part, the pulp. Moreover, the contact surface between this seed and the pulp is, in practice, the deepest point that can be reached in the fruit and it performs the role of a "thermal center" which in homogeneous solid objects is represented by the geometric center. According to Cinquanta et al. (2002) and Di Matteo et al. (2000, 2002, 2003), the theoretical solution is easy to deduce, following the methodology described by Carslaw and Jaeger (1959), to which should be added the works by Ruiz-López et al. (2004, 2007), which proposed analytical solutions for food drying kinetics, or the work by Helal (2012), who proposed an integral transform method for nonlinear heat-conduction problems in multilayered spherical media. Apart from these analytical works, we must resort