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Fluid–structure interactions in a cylindrical layered wave guide with application in the spinal column to syringomyelia[☆]

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ABSTRACT

Syringomyelia is a disease of the spinal cord in which fluid-filled cavities, called syrinxes, form and expand, compressing the surrounding neural tissue and producing neurological damage. This condition can occur following spinal injury and has limited treatment options in part because of a lack of understanding of its origins. Current pathogenic theories make predictions from localised disturbances in the cerebrospinal fluid dynamics. The poroelastic spinal cord tissues have an effective compressibility as a result of the localised displacement of the intercellular fluid. Also, despite the disease progressing over months and years, the mechanical perturbations from the heart and breathing cycles and from coughs and sneezes operate over much shorter timescales of seconds down to fractions of a second, respectively. To model these pathological features, we solve the harmonic eigenvalue problem for a two-dimensional elastic fluid–solid cylindrically layered waveguide. We analyse the dispersion behaviour and mode shapes over a wide wavelength spectrum and investigate the influence of Poisson's ratio on the wave mode characteristics. The healthy model consists of an elastic solid cylinder (spinal cord) and a surrounding annulus of inviscid fluid (cerebrospinal fluid in the subarachnoid space). To model syringomyelia we then add a cylindrical fluid space (syrinx) within the elastic cylinder. For the geometry with the inner fluid conduit present (diseased spinal cord in situ), the dispersion diagrams show three modes (0, 1, 2) that are propagatory across the wavelength spectrum. At long wavelengths, axial displacement dominates over radial, normal stress dominates over shear, but these become of the same order of magnitude as the wavelength approaches the radius of the elastic cylinder (spinal cord). Wave modes 0 and 1 induce relative motion between the elastic cylinder and the fluid contained within, which supports Williams's so-called slosh mechanism for syrinx lengthening, and all three modes involve stress concentrations (indicated by localised gradient peaks) in the inner wall of the elastic cylinder adjacent to the contained fluid, which would tend to support radial syrinx expansion. Mode 2 is the most sensitive to changes in Poisson's ratio (spinal cord tissue compressibility). We also investigate the progression of post-traumatic syringomyelia by simulating a sequence of discrete states in the gradual occlusion of the subarachnoid space and the subsequent formation and radial expansion of a syrinx. We predict that syrinx-fluid sloshing, hence the stress so induced, diminishes with the continued radial expansion of the syrinx. The largest normal stress concentrations at the syrinx boundary, which appear for modes 0 and 1, initially diminish with radial syrinx expansion; however, they

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reach a minimum value for intermediate syrinx diameters. On this basis we hypothesize that syrinx development may be part of a homeostatic mechanism to alleviate stress in the spinal cord. Understanding this process may aid in treatment development. The present work also has applications in industrial systems and serves as a platform for more advanced modelling of cylindrical waveguides in biological systems.

1. Introduction

1.1. Problem overview

This paper treats a fundamental problem in fluid–structure interaction (FSI), the dynamics of a cylindrical fluid–solid layered waveguide, which was inspired by and has application to the spinal disease syringomyelia. We study the wave-propagation characteristics of the spinal anatomy using the axisymmetric models depicted in Fig. 1(a) and 1(b), which contain, respectively, cross-sectional representations of the healthy spinal cord (SC) and a diseased SC with a fluid-filled cavity, termed a syrinx. We elucidate the dispersive properties of the spinal canal that have received little attention to date. Syringomyelia is a challenging condition to treat, with many patients developing progressive neurological disability that often does not improve with treatment. The present results are relevant, in particular, to the interpretation of measurements of CSF pulse waves and to understanding their potential role in syrinx development in post-traumatic syringomyelia. Understanding the cause will help improve treatment. Cylindrically-layered fluid–solid structures are a physiological building block, e.g., the arteries in the cardiovascular network and the ureter in the renal system, which may be augmented through pathologies (e.g., atherosclerosis) and associated treatments (e.g., catheter). More broadly, the analysis of the basic waveguide will also find applications in engineered systems.

Fig. 2 is the spinal anatomy represented by the engineering model depicted in Fig. 1(a). As shown, the soft SC lies within the protective confines of the bony canal formed through the vertebrae. Occupying the intervening annular space are the pia mater membrane, adhered to the cord surface, the cerebrospinal fluid (CSF) of the surrounding subarachnoid space, the bounding arachnoid mater and dura mater membranes, along with a collection of blood vessels (omitted in figure), nerves, fat and the ligaments that suspend the cord in place. The spine constitutes part of the wall that surrounds the thorax and abdomen. The epidural veins form a pair of channels that run the length of the anterior extradural aspect of the SC, and are connected to the veins of the spinal vertebrae, the attached muscles and the abdominal cavity. Any pressure perturbation in the thorax and abdomen will thus be transmitted to the spinal canal via these epidural veins.

We are concerned with wave propagation in our cylindrical models, which may be excited by a fluid pressure impulse. The pressure rise generated in the thorax and abdomen during coughing, or any similar percussive manoeuvre (sneezing, laughing, yawning, crying, valsalva), compresses the veins in the major body cavities, which displaces blood into the spinal epidural veins thereby transmitting most of the pressure rise along with it. As the contents of the spinal canal are essentially fluid-filled valveless flexible tubes, epidural pressure impulses are transmitted to the CSF, cord and membranes as waves having speeds determined by the spinal dynamic compliance. In Williams's human coughing experiments (Williams, 1972, 1976; Lockey et al., 1975), the so-induced CSF pressure impulses were of approximately 1 s duration with an abdominal pressure rise of 10–13 kPa (75–100 mm Hg) and had speeds of around 4.5 m/s. The ejection airspeeds of coughs and sneezes themselves were shown recently to reach similar values (Tang et al., 2013).

We now look at the progress that has been made in mathematical modelling of the spinal canal and the limitations that persist, which provides the rationale for the present approach.

1.2. Background

1.2.1. General trends

Table 1 lists all mathematical modelling papers published in peer-reviewed journals and edited books related to syringomyelia that model the flow of spinal CSF and intra-cord fluid using a continuum approach. These models have been categorized by their physics and mathematics, and within each category are ordered chronologically. Two-way fluid-structure interaction (2w-FSI) refers to the fully coupled solution of a fluid flow and the deformation of a flexible solid. One-way fluid-structure interaction (1w-FSI) models also have both fluid and solid motion but the latter is prescribed; i.e., the solid affects the fluid but not vice versa. The third physics category contains the computational fluid dynamics (CFD) models, which solve fluid flow through a static rigid geometry. There have been several studies published on the fluid mechanics at the craniocervical junction (e.g., Hentschel et al., 2010; Linge et al., 2011, 2014; Shaffer et al., 2014). Although these model the congenital condition Chiari malformation, which is found in around three quarters of all cases of syringomyelia (Williams, 1990), the fluid mechanics of the spinal canal itself was not in their scope, hence their omission from Table 1.

In the mathematics, we differentiate between analytical (Analyt) and numerical (Num) solutions; see Table 1 caption for an explanation of the various solution methods. The number of spatial dimensions (Dim) of the models are listed and a superscripted '∞' indicates that one of these dimensions (usually along the spinal axis) is infinite; i.e., there are no end conditions. The model geometries are either (i) idealized (1D, 2D, 3D), such as in a system of axisymmetric tubes, (ii) simplified (2D, 3D), in which anatomical-like