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Exact solution of axial liquid-pipe vibration with time-line interpolation

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ABSTRACT

The axial vibration of liquid-pipes, considering the fluid-structure interaction (FSI), can be described by a four-equation model consisting of two equations for the fluid and the other two for the pipe wall. When distributed friction and damping were neglected, the model could be solved with an exact solution developed from the method of characteristics (MOC). The exact solution with a simple recursion process was presented instead of meshing the distance-time plane, introducing no numerical error, while the only weakness was the high time cost. An improved method based on this exact solution is proposed in this work, using the time-line interpolation rather than the recursive algorithm to speed the calculation. As numerical diffusion and oscillation affect the accuracy a lot, various interpolation techniques are investigated and compared. The mean absolute error (MAE) and the structural similarity index measure (SSIM) are employed to assess the accuracy of proposed methods, and the latter was originally developed for the image quality assessment. A hybrid interpolation scheme, mixing the cubic spline and linear interpolation, is proposed and achieves the highest accuracy and the quadratic-linear interpolation is found to be a good choice considering both accuracy and efficiency. The solution method developed here improves the efficiency significantly while retains an acceptable level of the accuracy, especially suitable for the long duration event.

1. Introduction

The fluid-structure interaction (FSI) effect in liquid-filled pipes is significantly considered in the fields of hydraulic systems, nuclear industry, and water supply systems. The FSI model, containing fluid equations (extended from classical water hammer theory) and structure equations (based on the beam theory), was developed for describing the coupling between the fluid and pipe wall. The axial vibration of the liquid-pipe refers to fluid transient, axial pipe vibration and fluid-structure coupling, which can be modelled with four hyperbolic partial differential equations (PDEs). This four-equation model was proposed by Skalak (1956), Tijsseling et al. (2008) and Thorley (1969), and its validity has been fully demonstrated (Walker and Phillips, 1977; Wiggert et al., 1985; Budny et al., 1991; Zhang et al., 1999).

The method of characteristics (MOC) is widely used to solve linear hyperbolic PDEs in the time domain. The MOC has been proved to be the most popular and efficient time-domain approach for fluid transients (Amein, 1966; Vardy, 1976; Wiggert and Sundquist, 1977; Goldberg and Wylie, 1983; Lai, 1988) and FSI problems (Burmann, 1975; Schwarz, 1978; Wiggert et al., 1985, 1987; Budny et al., 1991; Tijsseling, 1993, 1996). An investigation (Ghidaoui et al., 2005) indicated that most commercially available

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Nomenclature

Uppercase letters

A	cross-sectional area
E	Young's modulus
K	fluid bulk modulus
L	length of pipe
M	number of interpolating points
N	length of the vector
P	fluid pressure
S	second derivative with respect to time, see Eq. (16).
V	fluid velocity

Lowercase letters

e	thickness of pipe wall
r	radii of pipe cross-section
t	time
u	pipe displacement
z	distance along the pipe
α	weight coefficient of the luminance in SSIM
β	weight coefficient of the contrast in SSIM
γ	weight coefficient of the structure in SSIM
ζ	interpolation factor
η	element of the vector $\boldsymbol{\eta}$
λ	eigenvalue, see Eq. (8).
ν	Poisson's ratio
ρ	density
σ	stress in pipe cross-section
Δt	time step

Subscripts

f	fluid
i	vector element index
p	pipe
z	axial coordinate
0	initial state ($t = 0$)

Superscripts

T	transposed
k	time level index, see Fig. 2.
n	time level, see Fig. 2.

Matrices and vectors

$\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$	coefficient matrix of FSI model
$\mathbf{D}$	boundary condition matrix
$\mathbf{O}$	zero matrix
$\mathbf{Q}$	excitation vector
$\mathbf{S}$	transformation matrix of eigenvectors, see Eq. (6).
$\boldsymbol{\eta}$	invariant vector, see Eq. (6).
$\boldsymbol{\phi}$	state vector of variables

water hammer software packages are based on the MOC. The principle of the MOC is to transform PDEs into ordinary differential equations (ODEs) along characteristic lines. The computation process is to mesh the distance-time plane, and time march from the initial conditions. The interpolation scheme (Goldberg and Wylie, 1983; Lai, 1988) and wave speed adjustment (Lavooij and Tijsseling, 1991; Tijsseling, 1993; Tijsseling et al., 1996) were employed, however, introducing numerical damping and phase error inevitably.

A number of researchers (Burmann, 1975; Williams, 1977; Wilkinson and Curtis, 1980) had made efforts on the exact time-domain solution of fluid transients and FSI, but these existing methods had limitations (Tijsseling, 2003). An exact solution of non-dissipative and non-dispersive wave propagation, without any interpolations and adjustments, was proposed by Tijsseling (2003, 2009; Tijsseling and Bergant, 2007), which could be implemented by means of simple recursions. The wave front was tracked backwards to the initial condition in time, and no computational grid was needed. It achieved exact results without errors of conventional methods. The exact solution can be used to verify the effectiveness of other time-domain methods and investigate their errors, while the only weakness is the high calculation time cost. As the recursion backwards to the initial condition were required at each time step, the calculation time would increase exponentially (Tijsseling, 2003). Attempts on the parallelisation of this time-consuming algorithm was made (Loh and Tijsseling, 2014), concluding that the speedup was linear in the number of computer processors whereas the computation time was exponential in simulation time, which means the calculation was still in exponential growth.

An improved exact solution based on Tijsseling's method is proposed in this work, using the time-line interpolation instead of recursions to speed the calculation of the FSI four-equation model. The commonly used linear interpolation may introduce the numerical diffusion, which could be seen more clearly in the event of longer duration (see Fig. 5). The present study is aim to seek a better interpolation approach to retain the accuracy level as high as possible. As higher-order interpolation schemes can reduce the numerical damping, both quadratic and cubic interpolation techniques are investigated in the paper.

To evaluate the accuracy of the proposed methods, the mean absolute error (MAE) and the structural similarity index measure (SSIM) are employed. The latter is originally developed for image quality assessment, and used in the present topic for the first time. The literature review of this method is presented in Section 4. The effectiveness and efficiency of present solutions are verified by the numerical testing problem in Section 5. The comparison of these interpolations, as well as the impact of event duration and time step, is discussed in Section 6.

2. Theory

The axial vibration can be described with a 4-eqution FSI model, containing two fluid equations (from the water hammer theory) and two structure equations (from the beam theory). The theoretical work is focused on the solution of this PDE system. Based on