Journal of Fluids and Structures 69 (2017) 154–173

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Flutter of rectangular simply supported plates at low supersonic speeds

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ARTICLE INFO

Keywords:
Panel flutter
Single mode flutter
Single degree of freedom flutter
Rectangular plate

ABSTRACT

Aeroelastic instability of skin panels, known as panel flutter, can occur in the form of coupled-mode or single-mode flutter. While the first type of flutter usually occurs in one eigenmode (composed of the first and the second natural modes in vacuum) and yields well-studied nonlinear limit cycle oscillations, the single mode flutter can occur in several simultaneously growing eigenmodes, leading to complex nonlinear panel dynamics, including different co-existing limit cycles, periodic and non-periodic higher-mode oscillations. Structural nonlinearity and linear aeroelastic growth mechanism play the major role in this dynamic.

While the linear panel flutter boundaries in the two-dimensional formulation have been studied in detail, there are only few investigations of the boundaries in the three-dimensional case. Since the linear growth mechanism plays an essential role in nonlinear oscillations, its comprehensive study is an important step toward understanding of complex dynamics of skin panels in the three-dimensional case. In this paper, we investigate the flutter boundaries of rectangular panels simply supported at all edges, and use potential flow theory to calculate the unsteady pressure. The problem is considered in two formulations: a series of rectangular plates, attached to each other, and a single rectangular plate. Flutter boundaries of the first four modes are calculated, and their transformations with the change of the spanwise plate width are studied in detail.

1. Introduction

Panel flutter is a phenomenon of self-exciting skin panel vibrations in flight vehicles moving at high speeds. Unlike wing flutter, usually it does not immediately yield the destruction of panels, but results in fatigue damage and rapidly decreases their lifetime. Although panel flutter was first observed during WWII, the first essential theoretical studies were conducted a decade later (Movchan, 1956, 1957). In these works, the Kirchhoff–Love model for panel dynamics and piston theory for unsteady flow pressure (Iliushin, 1956; Ashley and Zartarian, 1956) were used. During the following decades, the panel flutter problem was studied in more complex formulations (Bolotin, 1963; Grigolyuk et al., 1965; Dugundji, 1966; Dowell, 1974; Novichkov, 1978; Mei et al., 1999; Algazin and Kiiko, 2006). In most of these works, the 'elastic' part of the problem was subject to complication: multi-layered and composite panels, non-flat shells, geometrical and material nonlinearity, and complex material properties, including viscoelastic materials, shape memory alloys, and piezoelectric materials (Kiiko and Pokazeev, 2005; Duan et al., 2003; Zhou et al., 1995). The 'aerodynamic' part of the problem was not changed: the piston theory was employed.

The linearised unsteady pressure of inviscid gas, acting on the oscillating plate (i.e., the potential flow theory), has the form of an

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http://dx.doi.org/10.1016/j.jfluidstructs.2016.11.014
Received 25 March 2016; Received in revised form 13 November 2016; Accepted 24 November 2016

integral operator of a combination of the plate deflection and its spatial derivative, with a kernel consisting of special functions (Miles, 1959). In the limit of $M \to \infty$, this expression yields the piston theory formula; however, for Mach numbers $M < 2$, the accuracy of the piston theory essentially drops, and it becomes totally invalid for $M < \sqrt{2}$. Substitution of the potential-flow expression for pressure into the plate equation yields an integro-differential eigenvalue problem, which, due to its complexity, was studied in just a few papers. Min-de (1958) and Ming-de (1984) gave a closed-form solution for the two-dimensional integro-differential problem, which makes the eigenvalue problem algebraic; however, the latter turned out to be so difficult that no attempts to solve it were made. Nelson and Cunnigham (1956), Dowell (1974), and Yang (1975) solved the same integro-differential equation numerically through Galerkin and finite element methods for certain parameter values. It was noticed that, along with coupled-mode flutter, which occurs when the problem is solved through the piston theory, calculations at $1 < M < \sqrt{2}$ show the presence of another flutter type, namely, single mode (also called single-degree-of-freedom) flutter. Cunnigham (1967) studied flutter of rectangular simply supported and clamped panels at M=1.3 through the potential flow theory and showed that single mode flutter is also present in the three-dimensional problem, but disappears if the plate becomes sufficiently narrow in the spanwise direction. Dowell (1974) also studied nonlinear limit cycle oscillations by combining a nonlinear von Karman plate model with linearised potential flow theory, and observed pure first-mode oscillations at $1 < M < \sqrt{2}$.

Later the problem of panel flutter was numerically analysed through more complex aerodynamic models, which take into account aerodynamic nonlinearity, or shear flow aerodynamics, or both. Bendiksen and Davis (1995), Bendiksen and Seber (2008), Mei et al. (2014), and Shishaeva et al. (2015) studied transonic and supersonic flutter in inviscid flow, while Dowell (1971, 1973), Gordnier and Visbal (2002), Hashimoto et al. (2009), Visbal (2014), and Alder (2015, 2016) investigated flutter in a viscous flow.

A completely different approach was used by Vedeneev (2005), who analytically studied the two-dimensional panel flutter problem with the potential flow aerodynamics through an asymptotic method of global instability (Kulikovskii, 1966) for sufficiently long plates. It was strictly proved that single-mode flutter exists, and the physical mechanism of perturbation growth was revealed. The important feature proved is that the single-mode flutter cannot be obtained if the piston theory is used (although, as was recently shown by Ganji and Dowell (2016), higher-order expansions of the potential flow theory in the oscillation frequency yield correct results at $M < \sqrt{2}$, while the piston theory, being the first-term expansion, is just wrong at these M). Later Vedeneev (2012, 2013a) solved this problem numerically and calculated the stability boundaries of the first six eigenmodes. Flutter region consists of coupled mode flutter region and single mode flutter regions in various modes; the single mode flutter is dominant at low supersonic Mach numbers and short plates. It was shown in a closed form (Vedeneev, 2007, 2013b) and confirmed numerically (Shishaeva et al., 2015) that the multiplicity of linearly growing eigenmodes at small supersonic speeds yields the complex nonlinear dynamics of the panel, which includes the co-existence of regular limit cycles and cycles with internal 1:2 resonance, higher-mode limit cycles, and non-periodic oscillations.

The asymptotic method of Kulikovskii (1966) was also effective in the investigation of the boundary layer influence: Vedeneev (2013c) and Bondarev and Vedeneev (2016) conducted a general study of the boundary layer effect on panel flutter for arbitrary boundary layer profiles and showed that this effect is very different for boundary layers over convex and concave walls, and can be essentially destabilising for certain flow conditions.

The three-dimensional flutter problem for rectangular panels of large lengths with potential flow aerodynamics was studied by Vedeneev (2006, 2010), using a modified asymptotic method (Kulikovskii, 2006). Formulation of the problem for a numerical solution without additional assumption of large plate length is more complicated than the two-dimensional problem because the integration area in the integro-differential operator for the unsteady pressure becomes two-dimensional in the shape of a triangle (Miles, 1959). In this formulation, the problem was studied only by Dowell (1974), who investigated first-mode flutter for three plate aspect ratios at $M < \sqrt{2}$, and by Cunnigham (1967), who considered several modes at Mach number M=1.3.

Thus, up to the present day, there is no general study of rectangular plate flutter in the potential-flow formulation for arbitrary aspect ratios and Mach numbers. The present paper aims to fill this gap, taking into account that linear plate dynamics is not only important by itself, but also plays a major role in the formation of nonlinear limit cycle oscillations (Shishaeva et al., 2015). We solve this problem in two formulations. First, we study a particular case; namely, we consider a series of simply supported rectangular plates attached to each other. In this case, the integral in the integro-differential eigenvalue problem becomes one-dimensional. Second, we solve the exact problem for an isolated plate and investigate the effect of the single panel. Flutter boundaries for the first four modes in the parameter space are studied in detail.

2. Formulation of the problem

We study the linear stability of an elastic plate which forms a part of the plane surface. One side of the surface is exposed to a supersonic gas flow, as shown in Fig. 1. The other side experiences constant pressure so that the undisturbed state of the plate is flat. The plate is simply supported along all edges. We consider two plate configurations. In the first one, the plate is an infinite strip of the chordwise length L_{xw}, which is periodically simply supported with the spanwise period L_{yw} (Fig. 1a). This also can be represented as an infinite series of rectangular plates of $L_{xw} \times L_{yw}$ size attached to each other. Obviously, due to connections between spans, they are all either simultaneously stable or unstable. In the second configuration, the plate is a single rectangle of $L_{xw} \times L_{yw}$ size (Fig. 1b).

While the plate equation and boundary conditions in both configurations are the same, the aerodynamics are different. In the first case, each rectangular segment is affected by surrounding segments; in the second case it is not. We will show that in the first