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Response analysis of uncertain structural-acoustic system based on multi-convex set model

Hui Yin, Dejie Yu*, Shengwen Yin, Baizhan Xia

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, Hunan 410082, People's Republic of China

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ABSTRACT

This paper presents the convex perturbation method (CPM) and a new non-probabilistic convex analysis method named Chebyshev convex method (CCM) for uncertain analysis of structural-acoustic system based on multi-convex set model. The uncertain properties of the structure domain and the acoustic domain are uncorrelated, thus it is reasonable to describe the uncertainties of the structural-acoustic system as a multi-convex set model rather than a single one. The well-known CPM with lower-order Taylor series expansions is limited to handle small convex uncertainties. To handle large convex uncertainties, the Chebyshev convex method (CCM) is proposed. In CCM, the Chebyshev polynomials for approximating the original function are obtained by treating the uncertain parameters described by the multi-convex set model as the corresponding marginal interval parameters; the bounds of the original function are calculated by applying the convex Monte Carlo simulation (CMCS) to the approximate function. Numerical results on two structural-acoustic systems verify that the accuracy of CCM is higher than that of CPM, and large convex uncertainties can be effectively handled by using the higher-order CCM with reasonable computational costs.

1. Introduction

In engineering practice, many factors such as manufacturing tolerances, aggressive environment effect and unpredictable external excitations may lead to uncertainties existing in the geometric dimensions, material properties, boundary conditions or other parameters of an engineering system. Dealing with these uncertainties effectively is very important in engineering design. The issue about how to deal with these uncertainties has got much attention in recent years. Generally, probabilistic method (Stefanou, 2009; Arwade and Grigoriu, 2004; Chryssanthopoulos and Poggi, 1995; Brunoa et al., 2009), fuzzy-set theory (Zadeh, 1965; Kala, 2005; Massa et al., 2006) and convex model (Ben-Haim and Elishakoff, 1990; Ben-Haim, 1993; Elishakoff and Elisseeff, 1994) are the main three methods to handle the parametric uncertainties in a system. If the object information of uncertain parameters is sufficient enough to construct the probability density function, probabilistic method is the prior way to handle the parametric uncertainties. Lots of techniques such as Monte Carlo method (Heaney and Cox, 2006; Spanos and Kontsos, 2008), spectral stochastic method (Adhikari, 2011; Chen and Guedes Soares, 2008) and perturbation stochastic method (Kamınski and Solecka, 2013; Cavdar et al., 2008) have been developed as the solution strategies for probabilistic model. Another way to handle the adequate uncertain information is using the fuzzy-set theory, in which the parametric uncertainties are described as fuzzy parameters whose membership functions can be defined unambiguously. The fuzzy finite element method (FFEM) (Rao and Chen,

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* Corresponding author.
E-mail address: djyu@hnu.edu.cn (D. Yu).

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1997) is a widely used approach for the numerical analysis of uncertain systems with fuzzy parameters, and by using the $\alpha -$ level technique (Moller et al., 2000), FFEM can be decomposed into a series of interval finite element method (IFEM).

Unfortunately, the uncertain information in many engineering practice is usually so limited that it is difficult to construct appropriate probability density functions or membership functions for the uncertain parameters, thus the non-probabilistic convex model comes into use. In the convex model, the uncertain domain is described as a convex set, thus only the boundaries of the uncertain domain are required. Since first proposed in the early 1990s, the convex model has been further developed by many researchers due to its capability to solve the uncertain problems with limited uncertain information (Jiang et al., 2011, 2013, 2015; Luo et al., 2008). The most widely used two types of convex models are the interval and the ellipsoid, in which the parametric uncertainty is assumed to belong to a multidimensional box and a multidimensional ellipsoid, respectively. Due to the simplicity and stability, the Monte Carlo method can also be a solution strategy for the convex model, but the computational cost is usually too high to be acceptable, especially for the analysis of large scale engineering systems. To improve the computational efficiency, Qiu et al. (1996) proposed an interval perturbation method (IPM) based on the truncated Taylor series and Neumann series for the interval convex model; Pantelides (1996) proposed a convex perturbation method (CPM) based on the truncated Taylor series and the Lagrange multiplier technique for the ellipsoidal convex model. Due to the high efficiency and acceptable accuracy, IPM and CPM have been widely used in the analysis of uncertain systems, such as the static, dynamic, and natural frequency analysis of uncertain structures (Qiu and Wang, 2005; Chen et al., 2009; Qiu et al., 2009), the uncertain acoustic field prediction (Xia and Yu, 2012, 2014). It is worth noting that both IPM and CPM are based on the Taylor series that are truncated, and usually only the first and second terms of Taylor series are retained, which is mainly because of the high-complexity of calculation of the objective functions' higher-order partial derivatives, thus the application of IPM and CPM are limited to the analysis of uncertain systems with small parametric uncertainties. The sub-interval technique (Qiu and Elishakoff, 1998; Xia et al., 2013a, 2013b) can enable the IPM to handle large interval parametric uncertainty by dividing a large interval into several small intervals, but this interval decomposition technique can't be applied to the CPM to handle large ellipsoidal parametric uncertainty. Therefore, a new effective method for handling large convex parametric uncertainty is promising.

In recent years, with the increase of the people's requirement for living environment, the noise pollution has received more and more attention. The analysis of the acoustic behavior of structural-acoustic systems can provide important information for noise control. The deterministic numerical analysis technique for structural-acoustic systems has achieved great success, such as the finite element method (FEM) (Zienkiewicz, 1977), boundary element method (BEM) (Allen and Vlahopoulos, 2000) and statistical energy analysis (SEA) (Lyon and DeJong, 1995). However, the uncertainties caused by various factors inevitably exist in a structural-acoustic system, thus uncertain analysis for structural-acoustic systems is absolutely necessary and meaningful in acoustic design. For the analysis of structural-acoustic systems with interval physical parameters, Xia and Yu (2013a, 2013b) developed a modified interval perturbation method (MIPM) by considering higher-order terms of the Neumann series, and it was proved that the accuracy of MIPM is better than that of IPM. Based on the work of Xia and Yu (2013a, 2013b), Wang et al. (2014) gave an interval analysis for structural-acoustic systems with uncertainties in both physical parameters and boundary conditions, and proposed a reliability-based optimization model for design of the structure domain. For the analysis of structural-acoustic systems with hybrid uncertain parameters, two methods were proposed: namely the hybrid vertex perturbation method (HPVM) (Xia et al., 2013a, 2013b) for uncertain analysis with random and interval parameters, the interval random perturbation method (IRPM) (Xia and Yu, 2013a, 2013b) for the uncertain analysis with interval random parameters. In a structural-acoustic system, parametric uncertainties may exist in both the structure domain and the acoustic domain, and the parametric uncertainties in the two separated domains can reasonably considered to be uncorrelated. Thus it is more reasonable to divide the parametric uncertainties of a structural-acoustic system into at least two independent groups, and describe them by using a multi-convex set model (Sun et al., 2013) rather than a single one. Up to now, study on the non-probabilistic model for structural-acoustic systems is mainly concentrated on the interval model, while the application of a more general convex model involving both the interval and the ellipsoid, which are of equal importance in engineering practice, is still unexplored. Thus it is promising to develop effective methods for uncertain analysis of structural-acoustic systems based on a multi-convex set model, which contains the interval and the ellipsoid. In a multi-convex set model, uncertainties are assumed to fall into multiple independent "multidimensional boxes" and (or) "multidimensional ellipsoids", and each "multidimensional box" or "multidimensional ellipsoid" is called as a sub-convex set in this paper. For the uncertain analysis based on a multi-convex set model, the first and second convex perturbation methods (FCPM, SCPM) are powerful techniques when the parametric uncertainty is small. However, as aforementioned, due to the high-complexity of calculation of the objective functions' higher-order partial derivatives with respect to the uncertain parameters, it is difficult to consider a higher-order CPM to handle large convex parametric uncertainty; besides, the sub-interval technique is only applicable for the interval model, while for the ellipsoidal model, this technique is incapable. Another strategy that can efficiently handle uncertainties is to use the surrogate models to approximate the original functions (Allaire and Willcox, 2010). There have been several kinds of surrogate models, such as the radial basic function (RBF) (Shan and Wang, 2010), Kriging (Zhang et al., 2015) and polynomial surrogate models (PSMs) (Fang et al., 2003; Liu et al., 2015). There is no one all-powerful surrogate model that suitable for all practical problems (Wu et al., 2016). The polynomial surrogate models may have advantages in transparency, efficiency, and conceptual simplicity over other models (Jin et al., 2001). However, for handling uncertainties, most of these surrogate models (RBF, Kriging, PSMs etc.) are concentrated on the probabilistic model, and few of them are applied to the non-probabilistic convex model.

In Refs. (Wu et al., 2013a) and (Wu et al., 2013b), a Chebyshev interval method (CIM) based on Chebyshev polynomials is proposed for the interval analysis of ordinary differential equation (ODE) systems and differential algebraic equation (DAE) systems, respectively. The CIM is essentially a technique that first applies the PSM to handle non-probabilistic interval uncertainties. Compared with IPM, CIM can control overestimation better, and thus achieve a higher accuracy. Besides, there is no need for CIM to