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Wave-induced hydrodynamic responses of a rigid body connected with elastic plates floating on a two-layer fluid

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ABSTRACT

A novel physical model of Very Large Floating Structures (VLFSs) is investigated for a two-layer fluid. This model is composed of a semi-immersed module rigidly connected with two elastic plates, which may provide basic knowledge for the study of multi-module floating structures in a stratified ocean. Under the hypothesis of small-amplitude wave theory, the case with the coupling effects among the wave motion, the vertical deformation of the elastic plates and the rigid body's oscillation is solved by only considering a scattering problem and a radiation one. New inner product relations with orthogonality are suggested to calculate the undetermined coefficients in eigenfunction expansions, which is eligible to yield a convergent result. By this approach, the matching relations are simplified and the numerical calculation speed is remarkably promoted. Exciting forces (caused by the scattering potential), hydrodynamic coefficients for added mass and radiation damping (caused by the direct act of radiation potential) and additional coefficients for added stiffness and added damping (caused by the indirect act of radiation potential via the elastic plates) are obtained. Numerical representations for amplitudes, exciting forces, hydrodynamic coefficients, and additional coefficients are shown, in which Green's theorem is used for ensuring convergence.

1. Introduction

A Very Large Floating Structure (VLFS) is an appropriate class of naval architecture which is premised on exploiting expansive ocean space for many particular purposes. The ratio of the structure's horizontal length to its vertical dimension is usually large enough, which implies the structure's flexible deformation will have significant influences on its overall response. Resembling to ice floes, VLFSs are usually modeled as thin elastic plates floating on the fluid surface without any cavitations or draft, and are formulated with the help of hydrodynamics and elasticity (Fox and Squire, 1990, 1994; Kashiwagi, 1998). In recent researches, investigations on multi-module VLFSs have become one of the most attractive research issues (Hirdaris and Temarel, 2009; Loukogeorgaki et al., 2012; Montiel et al., 2012; Hirdaris et al., 2014). The realization of more complex and improved structures depends on the further development of hydroelastic theory. An immersed rigid body connected with elastic plates by rigid joints may be a representative physical model for multi-module VLFSs. The body supports certain buildings, for example, living facilities, oil storages, airport terminals, on the VLFS. Due to the special characteristics of this typical structure, the hydrodynamic responses will be affected vigorously by the plate's elastic deformation and the rigid body's oscillation.

Waves are one of the most common excitations to the VLFS in real ocean conditions, which are related to the complicated

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properties of sea water such as stratification on density. Linton and Cadby (2002) considered oblique incident waves scattered by a horizontal cylinder in a fluid with an upper layer lying on an infinitely deep layer with greater density. By using the linear water wave theory and the multipole expansion method, Linton and Cadby (2002) found that waves could propagate at two different wave numbers, which refer to the surface and interfacial modes, and established the conservation relation of energy. Rusås and Grue (2002) investigated solitary waves propagating in a three-layer fluid and they found that the amplitudes become larger compared to those in a two-layer fluid, where the thickness of density pycnocline is neglected. Bhattacharjee and Sahoo (2008) derived the general formulae of velocity potentials for the flexural–gravity waves in two-layer fluids of infinite and finite depths. They applied the eigenfunction expansions to study the impacts of a line source on the flexural–gravity waves and investigated the wave scattering by a narrow crack in an infinite ice sheet. The conclusions are that the reflection and transmission coefficients are obviously affected by fluid stratification while the presence of a crack had negligible influences on the propagation of internal waves.

As typical shapes combining modular VLFSs, many simple objects, e.g. cylinder, cuboid and plate, have been studied in many previous investigations for their hydrodynamic characteristics. Referring to variational principle, Mei and Black (1969) studied the problem of wave scattering by rectangular obstacles either mounted on the seabed or keeping stable on the surface. Later, Black et al. (1971) applied the method of Mei and Black (1969) to consider the radiation case of waves undergoing a rigid body's oscillation in small magnitude. Abul-Azm (1993) found that the method of matched eigenfunction expansions is also appropriate for the calculation of immersed objects, and they investigated the wave transmission and reflection for thin, rigid and impermeable breakwaters submerged at arbitrary water depths. Wu et al. (1995) investigated the wave-induced responses of a floating elastic plate by expanding the wave's velocity potential and the deformable plate's motion in terms of the vertical eigenfunctions and the modal functions, respectively. Ohkusu and Namba (2004) used an analytical approach to study the bending vibration of a rectangular thin elastic plate floating in shallow water, and subsequently obtained an explicit representation under the frame of linear shallow water theory.

In order to obtain better hydrodynamic performances and provide eligible functionality, various kinds of attachments (e.g. breakwater, living building) are usually designed for multi-module VLFSs. The fluid–structure interaction will become more sophisticated due to the strong coupling effect between wave motions and the combined structure. Paulling and Tyagi (1993) analyzed the responsive motion of a multi-module VLFS composed by a concatenation of many rigid bodies with six degrees of freedom. Wu et al. (2004) investigated the hydrodynamic coefficients for a cylinder floating over a caisson mounted on seabed, which can be regarded as an abstract representation for a wave power device. Pham et al. (2008) modeled a circular VLFS connected with a submerged horizontal annular plate, and they also considered the effectiveness of the attachment in reducing the dynamic responses of structure. Riyansyah et al. (2010) studied a VLFS composed of two thin elastic plates articulated by an elastic joint, and tried to find the optimized connector location as well as stiffness for better hydrodynamic performance. Lin and Lu (2014) investigated the wave diffraction for a combination case of an annular elastic plate connected with a bottom-mounted cylinder, and discussed the forces exerted on the cylinder subsequently.

The method of matched eigenfunction expansions is a widely used technique for the hydrodynamic interaction between waves and a marine structure. The key point in this method is to obtain the unknown coefficients in the expansions. A usual approach for calculating the numerical coefficients is the error function method, where the potential functions are fitted along the matching boundary by minimizing the errors with the help of the least square technique. This approach was presented by Fox and Squire (1990,1994) who investigated the hydrodynamic responses due to the wave scattering by a semi-infinite ice floe. Sahoo et al. (2001) adopted an inner product method to calculate the same problem. They employed the vertical eigenfunctions of flexural–gravity waves to define a new-type inner product, and made it orthogonal by adding a differential term. Conclusively, the calculation process was simplified in some degree, because the inner product method is much easier to manipulate and is independent of empirical Lagrangian multipliers in comparison with the error function method. Xu and Lu (2010) improved the convergence rate of the inner product method for calculating the same case as (Sahoo et al., 2001). They employed the vertical eigenfunctions in the free-surface region to make an inner product and found that the new-type inner product is automatically orthogonal. By the orthogonal relation, the numerical approximation for the series solution can converge quickly after truncating a few terms in the expansions.

In the present study, we investigate the hydrodynamic responses of an immersed body connected with two thin elastic plates rigidly, which can be thought of as a multi-module VLFS. Under the small-amplitude wave hypothesis and the linear potential flow theory, we consider the model by a linear superposition of a scattering potential and a radiation one. Subsequently, the problem is solved by using the inner product method. The technical difficulty is to have the fluid velocities well matched for the fluid boundary between the elastic plate region and the rigid body region. The vertical eigenfunctions underneath the elastic plate are invalid to make an inner product, because their completeness has never been proved before and could “lead to an ill-conditioned system of equations” (Kohout et al., 2007). Considering that the free-surface wave is a limiting case of the flexural–gravity wave, we try to employ the vertical eigenfunctions in the free-surface region to make an inner product. The effectiveness of this approach is found and some orthogonal relations which are helpful for simplifying the matching equations will be yielded afterwards. The computational efficiency is promoted. By taking coordinate transformations, a new physical explanation for the complex wave numbers of flexural–gravity wave has been given; this explanation may be a progressive understanding for wave scattering by elastic plates. The numerical results about the wave amplitudes, exciting forces, coefficients of added mass, radiation damping, added stiffness, and added damping are exhibited, where Green's theorem is employed for ensuring convergence.

2. Mathematical formulation

We consider a two-dimensional model composed of a semi-immersed rigid body and two pieces of thin elastic plate floating on a