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Extreme nonlinear energy exchanges in a geometrically nonlinear lattice oscillating in the plane

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We study the in-plane damped oscillations of a finite lattice of particles coupled by linear springs under distributed harmonic excitation. Strong nonlinearity in this system is generated by geometric effects due to the in-plane stretching of the coupling spring elements. The lattice has a finite number of nonlinear transverse standing waves (termed nonlinear normal modes – NNMs), and an equal number of axial linear modes which are nonlinearly coupled to the transverse ones. Nonlinear interactions between the transverse and axial modes under harmonic excitation give rise to unexpected and extreme nonlinear energy exchanges in the lattice. In particular, we directly excite a transverse NNM by harmonic forcing (causing simulataneous indirect excitation of a corresponding axial linear mode due to nonlinear coupling), and identify three energy transfer mechanisms in the lattice. First, we detect the stable response of the directly excited transverse NNM (despite its instability in the absence of forcing), with simultaneous stability of the indirectly excited axial linear mode. Second, by changing the system and forcing parameters we report extreme nonlinear "energy explosions," whereby, after an initial regime of stability, the directly excited transverse NNM loses stability, leading to abrupt excitation of all transverse and axial modes of the lattice, at all possible wave numbers. This strong instability is triggered by the parametric instability of an indirectly excited axial mode which builds energy until the explosion. This is proved through theoretical analysis. Finally, in other parameter ranges we report intermittent, intense energy transfers from the directly excited transverse NNM to a small set of transverse NNMs with smaller wavelengths, and from the indirectly excited axial mode to a small set of axial modes, but with larger wavelengths. These intermittent energy transfers resemble energy cascades occurring in turbulent flows. Our results show that nonlinear mechanical systems can support extreme energy transfer phenomena, with features resembling "mechanical turbulence".

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1. Introduction

Energy transfer is a central problem in dynamical systems, and the main focus in many applications, ranging from vibration isolation concerned with reducing the energy flux into a structure, and vibration absorption seeking to remove energy from a system, to energy harvesting aiming at transferring energy from the mechanical to the electrical domains within a system based on the balance between the energy distribution in the system and the rate at which this energy is harvested. A central feature of linear systems is that energy can be segregated in their linear normal modes, with no possibility of energy transfer between them. In contrast, while the response of nonlinear systems can be described in terms of the vibration modes identified from a linearized analysis, these modes are no longer decoupled and nonlinearity allows for energy transfers between them. Moreover, under certain conditions (Vakakis et al., 1996), *nonlinear normal modes – NNMs* can be defined for which energy is again segregated, allowing for an approximate description of the system dynamics in terms of isolated (non-resonant) nonlinear modes. NNMs were defined as synchronous standing waves of a nonlinear system, resembling the classical linear normal modes. Understanding and passively controlling how energy is transferred between coupled systems or NNMs, and across temporal or length scales within a system is central to the dynamical analysis and design of mechanical systems.

Perhaps the most famous example involving intense nonlinear energy transfers are those occurring between modes/scales in turbulent fluid flows. From an energy transfer point of view turbulent flows have two very important properties: They are robust and almost impossible to suppress, and they dissipate large amounts of energy compared to laminar flows of the same energy. The reason is that in a typical turbulent flow energy is transferred through a nonlinear mechanism over a large number of modes (broadband spectrum) resulting in simultaneous dissipation over a large number of scales (Barenblatt, 1983). In turbulent systems with quadratic nonlinearities, the finite sizes of high-dimensional chaotic attractors are caused exclusively by the synergistic activity of persistent, linearly unstable directions and a nonlinear energy transfer mechanism in non-linearizable quadratic oscillators (Sapsis, 2013; Sapsis and Majda, 2013).

In mechanical systems intense energy transfer phenomena have been observed also in the form of targeted energy transfers from primary structures to strongly nonlinear attachments (Vakakis et al., 2008). The dynamics and energy transfer phenomena in phononic lattices are reviewed by Hussein et al. (2014). Aubry (1997) and Flach and Gorbach (2008) reviewed energy transfer and focusing phenomena in nonlinear Hamiltonian lattices through propagating discrete breathers. Energy transfers in the form of traveling waves and breathers in neural networks are studied in Folias and Bressloff (2005), whereas Starosvetsky et al. (2012) and Hasan et al. (2013) studied strong energy exchanges and irreversible energy transfers in weakly coupled, strongly nonlinear one-dimensional granular chains mounted on elastic foundations. Zhang et al. (2015) demonstrated passive pulse redirection and nonlinear targeted energy transfer in weakly coupled granular networks under harmonic excitation. Smirnov et al. (2013) investigated the energy transfer exchange phenomenon in the finite nonlinear Fermi-Pasta-Ulam (FPU) lattice. It was found that resonant interactions between high-frequency nonlinear normal modes (NNMs) lead either to complete energy exchanges between different parts of a lattice, or to energy localization in a subset of the lattice.

In this work we study the forced and damped in-plane oscillations of a nonlinear lattice composed of a finite number of particles coupled by linear springs and viscous dampers under harmonic excitation. This lattice was first introduced by Manevitch and Vakakis (2014), and its strong geometric nonlinearity is generated by the stretching of the linear coupling springs during the in-plane particle oscillations. For fixed boundary conditions the lattice supports almost linear sound waves corresponding to predominantly axial oscillations of the particles, as well as strongly nonlinear waves corresponding to predominantly transverse oscillations of the particles. Moreover, in the low-energy limit this lattice behaves as a nonlinear sonic vacuum, i.e., a medium with zero speed of sound as defined in the context of classical acoustics. The complete absence of linear acoustics enables strongly nonlinear resonance interactions between standing transverse waves of the lattice (NNMs), as well as strongly nonlocal effects in the nonlinear acoustics. Other recent works reported nonstationary resonance effects in the unforced lattice, in the form of intense recurring energy exchanges between different parts of the lattice (Koroleva and Manevitch, 2015; Koroleva et al., 2015), and accelerating oscillating fronts with localized envelopes (Gendelman et al., 2016). Moreover, in (Zhang et al., 2016) interesting non-reciprocal effects in the dynamics and acoustics were studied, including energy exchanges from small-to-large length scales; wave interaction phenomena in the form of irreversible targeted energy transfers from axial linear waves to nonlinear transverse pulses; and complex energy cascading processes. Similar non-reciprocity phenomena have been observed even in conservative continuum models with non-uniformly distributed system parameters, in the form of unidirectional broadening of the corresponding vibration spectrum (Cousins and Sapsis, 2014).

By harmonically forcing a single transverse NNM of the lattice and examining the ensuing intense energy transfer phenomena, we aim to show that unexpected and extreme energy transfers can be realized. These include "energy explosions" and intermittent energy exchanges between the directly excited NNM and finite subsets of transverse and axial modes with larger or smaller wavelengths. The reason for these complex energy transfers is the strong geometric nonlinearity of the lattice, which leads to dynamical interactions between the directly excited transverse NNM and the other indirectly excited transverse and axial modes. Perhaps this is surprising given the relatively simple configuration of the lattice, but it can be explained by the sonic vacuum limit that this system reaches for low energies, and the strong nonlocality of the dynamic interactions that occur when that limit is reached (Manevitch and Vakakis, 2014). Both stationary and nonstationary