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Modeling fibrous biological tissues with a general invariant that excludes compressed fibers

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ABSTRACT

Dispersed collagen fibers in fibrous soft biological tissues have a significant effect on the overall mechanical behavior of the tissues. Constitutive modeling of the detailed structure obtained by using advanced imaging modalities has been investigated extensively in the last decade. In particular, our group has previously proposed a fiber dispersion model based on a generalized structure tensor. However, the fiber tension–compression switch described in that study is unable to exclude compressed fibers within a dispersion and the model requires modification so as to avoid some unphysical effects. In a recent paper we have proposed a method which avoids such problems, but in this present study we introduce an alternative approach by using a new general invariant that only depends on the fibers under tension so that compressed fibers within a dispersion do not contribute to the strain-energy function. We then provide expressions for the associated Cauchy stress and elasticity tensors in a decoupled form. We have also implemented the proposed model in a finite element analysis program and illustrated the implementation with three representative examples: simple tension and compression, simple shear, and unconfined compression on articular cartilage. We have obtained very good agreement with the analytical solutions that are available for the first two examples. The third example shows the efficacy of the fibrous tissue model in a larger scale simulation. For comparison we also provide results for the three examples with the compressed fibers included, and the results are completely different. If the distribution of collagen fibers is such that it is appropriate to exclude compressed fibers then such a model should be adopted.

1. Introduction

Constitutive models of fibrous soft biological tissues that have been proposed to account for the underlying microstructure have been employed extensively to simulate the mechanical response of the tissues (see, e.g., Holzapfel et al., 2015; Holzapfel and Ogden, 2010), and to inform the development of new medical devices (Mao et al., 2016). The latest advances in imaging techniques have revealed details of the microstructure of biological tissues such as arterial walls (Niestrawska et al., 2016; Schriefl et al., 2012, 2013). Fiber dispersions have been observed not only in arterial walls but also in articular cartilage (Ateshian et al., 2009; Lilledahl et al., 2011), carotid arteries (Azinfar et al., 2017), the myocardium

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(Covell, 2008; Karlon et al., 1998), the pericardium (Sacks, 2003), and other tissues. In particular, the knowledge of layer-specific three-dimensional (3D) dispersion of collagen fibers embedded in the ground substance of tissues provides a better understanding of the underlying mechanism of tissue mechanical behavior and facilitates the development of new constitutive models.

The mathematical description of fiber dispersion in a constitutive equation for computational simulations of fibrous tissues poses formidable challenges even with considerable idealizations and simplifications. Since the pioneering work of Lanir (1983) on the angular integration (AI) approach for incorporating fiber dispersion in a strain-energy function there have been numerous studies based on this approach; see, e.g., the review article of Holzapfel and Ogden (2015) and references therein, in addition to more recent works such as Li et al. (2016, 2017). Although the physical interpretation of the AI approach is clear and easy to understand, its computational implementation requires numerical integration over a spherical domain at each Gauss point during a finite element analysis, which is computationally expensive. When exclusion of compressed fibers is considered, the AI approach requires even more computational time, which could be reduced by using a high-performance computing cluster (Li et al., 2017).

By contrast the generalized structure tensor (GST) approach (Gasser et al., 2006) requires much less computational time, and recently this approach has been shown to be equivalent in predictive power to that of the AI approach (Holzapfel and Ogden, 2017). In passing we note that it has recently been brought to our attention that a notion equivalent to our generalized structure tensor was introduced (much) earlier in the context of the rheology of short fiber composites by Advani and Tucker (1987). The GST approach has been used extensively in recent years and is based on a so-called generalized structure tensor $\mathbf{H}$ defined by

$$\mathbf{H} = \frac{1}{4\pi}\int_{\mathbb{S}^2} \rho(\Theta, \Phi)\mathbf{N}\otimes\mathbf{N}\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi, \tag{1}$$

where Θ and Φ are two spherical polar angles, $\mathbb{S}^2 = \{(\Theta, \Phi) \mid \Theta \in [0, \pi], \Phi \in [0, 2\pi]\}$ denotes a unit sphere, and the probability density function (PDF) $\rho(\Theta, \Phi)$ represents the relative probability density of fibers at an arbitrary orientation $\mathbf{N}$ around a mean direction $\mathbf{M}$ in the reference configuration of the tissue. The PDF can be determined from imaging data of the fiber distribution in the tissue, and the PDF is normalized according to

$$\frac{1}{4\pi}\int_{\mathbb{S}^2} \rho(\Theta, \Phi)\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi = 1. \tag{2}$$

In addition, a Green–Lagrange strain-like quantity E was introduced as

$$E = \frac{1}{4\pi}\int_{\mathbb{S}^2} \rho(\Theta, \Phi)I_4(\mathbf{N})\sin\Theta\,\mathrm{d}\Theta\,\mathrm{d}\Phi - 1, \tag{3}$$

where $I_4 = \mathbf{N}\cdot\mathbf{CN}$ is a pseudo-invariant (Holzapfel, 2000), which is equal to the square of the fiber stretch in the direction $\mathbf{N}$, and $\mathbf{C}$ is the right Cauchy–Green tensor. This quantity E was used in the strain-energy function introduced in Gasser et al. (2006), which is now referred to as the GOH model.

In the original work (Gasser et al., 2006), it was stated that the fibers would contribute to the strain–energy function via $\mathbf{H}$ when the strain in the mean fiber direction $\mathbf{M}$ is positive. However, for computational purposes this condition was implemented as $\bar{E} > 0$, where $\bar{E}$ is defined in (3), with I_4 replaced by its isochoric counterpart $\bar{I}_4 = \mathbf{N}\cdot\bar{\mathbf{C}}\mathbf{N}$, where $\bar{\mathbf{C}} = (\det\mathbf{C})^{-1/3}\mathbf{C}$. In the nonlinear finite element program ABAQUS (Dassault Systèmes Simulia Corporation, 2017) the GOH model is implemented by using $\bar{E} > 0$ for the switch. This leads to continuous stresses and their derivatives, whereas a switch based on the strain in the mean fiber direction may lead to discontinuous stresses and derivatives. When the mean fiber direction in a dispersion is extended then in general some of the fibers in the dispersion will be compressed and such fibers are not excluded by the GOH model. For an incompressible material it is always the case that some fibers are compressed when others are extended and *vice versa*.

It is not surprising that this rather ‘abrupt’ treatment leads to a discontinuous stress response as revealed in the recent study (Latorre and Montáns, 2016) since the authors misinterpreted the GOH model. In that study, an equivalent transversely isotropic deformation state was defined by using squared stretches in the mean fiber direction and an average of the squared stretches of all the fibers in the plane transverse to the mean direction to exclude compressed fibers from a dispersion, and thus a continuous stress response is achieved. However, as the authors mentioned, if both the squared stretches are greater than one, then no exclusion of the compressed fibers is possible. Indeed, for simple shear, it is straightforward to show that this tension-compression criterion for dispersed fibers does not exclude all the compressed fibers when the amount of shear is large. Thus, this approach may be only applicable for some (rather) special cases. This motivates the need for a physically realistic switch for the GOH model which avoids discontinuities. It has been stated several times in the literature that it is not possible to exclude compressed fibers within the GOH approach, but this is not the case, as was recently shown in Holzapfel and Ogden (2017).

In this paper we provide an alternative approach to the exclusion of compressed fibers on a quite different basis to the one in Holzapfel and Ogden (2017). Again we consider $\rho(\Theta, \Phi)$ to satisfy the normalization condition (2) and it follows from (3) that $E = 0$ for deformations for which $I_4 \equiv 1$, i.e. in the reference configuration.

The present study is structured as follows. In Section 2 we describe the form of a new general invariant that excludes fibers under compression. Then, based on this new invariant, we present a new strain-energy function in which the total