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A family of hyperelastic models for human brain tissue

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ABSTRACT

Experiments on brain samples under multiaxial loading have shown that human brain tissue is both extremely soft when compared to other biological tissues and characterized by a peculiar elastic response under combined shear and compression/tension: there is a significant increase in shear stress with increasing axial compression compared to a moderate increase with increasing axial tension. Recent studies have revealed that many widely used constitutive models for soft biological tissues fail to capture this characteristic response. Here, guided by experiments of human brain tissue, we develop a family of modeling approaches that capture the elasticity of brain tissue under varying simple shear superposed on varying axial stretch by exploiting key observations about the behavior of the nonlinear shear modulus, which can be obtained directly from the experimental data.

What we know to be true and what we believe to be reasonable for one or another real material serve as our guides in choosing different forms of constitutive equations. – Clifford Truesdell (1966).

1. Introduction

The study of the mechanical response of biological systems within a continuum framework relies on constitutive equations relating stresses to strains (Holzapfel, 2000). In the absence of a method to derive these constitutive equations from first principles, phenomenological models are routinely used. In particular, when a system behaves in the elastic regime, classes of hyperelastic models have been proposed for many tissues and organs. Ideally, these models are systematically calibrated and validated on multiaxial loading data (Misra et al., 2010; Sommer et al., 2013). Rather than using brute force and fit data to arbitrary strain-energy functions, it is well understood that a key element of constitutive modeling is to consider families of models with desirable properties. For instance, collagen-rich soft tissues are known to be mostly incompressible and display strong strain-stiffening response. Therefore, most of the current models for these tissues start with a functional form that both enforces these particular properties and is general enough to be adapted for specific systems.

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Brain tissue is strikingly different from most soft biological tissues: its microstructure is not governed by collagen and elastin fibers, which implies that brain typically lacks the characteristic strain-stiffening behavior of arteries, skeletal and cardiac muscle, or skin (Goriely et al., 2015a; 2015b). The typical behavior of these tissues, captured by models such as Fung's or Gent's (Horgan and Saccomandi, 2003), is that a strong stiffening is obtained at finite extension leading either to a singular limit (in the case of the Gent model) or exponential behavior (for the Fung model). Data analyses shows that these models are not suitable for brain tissue (Mihai et al., 2015). A natural problem is then to understand the defining characteristics of brain tissue and to identify a suitable family of hyperelastic models with these characteristics. Moreover, a model for brain tissue needs to be suitable for small to moderate strain as experienced in vivo (Bayly et al., 2006).

The analysis presented in this paper is based on the data of human brain tissues tested under finite uniaxial and multiaxial loading reported in Budday et al. (2017). In this article, the authors have established that the microstructural anisotropy due to the alignment of nerve fibers in the tissue does not result in an anisotropic elastic response similar to the effect of collagen fibers in other soft tissues. Therefore, we neglect a possible anisotropic response and assume here that brain tissue is isotropic. Another important consideration is the viscoelastic response of brain tissue. The data shows clearly that the response of brain tissue has a viscous component indicated by a different response in loading and unloading. However, in the first instance, we are interested in the tissue's effective elastic response under small strain rate. Following Budday et al. (2017), this response is obtained as the average between the loading and the unloading paths, assuming that this corresponds to the case when the strain rate approaches zero and the hysteresis vanishes. Therefore, for the rest of our analysis, we restrict our attention to isotropic elastic models.

In the elastic regime, recent experiments on brain tissues have further established another response under combined compression and shear, namely that the elastic shear stress increases sharply with increasing axial compression, while it only increases moderately with increasing tension (Budday et al., 2017; Levental et al., 2007; Mihai et al., 2015; Pogoda et al., 2014). From a modeling point of view, capturing this apparently contradictory behavior represents a major challenge. Similar behaviors were also observed in other soft tissues with large lipid content, such as adipose tissue (Mihai et al., 2015).

For brain and adipose tissues, Ogden-type hyperelastic incompressible isotropic models (Ogden, 1972) have been found in good agreement with the experimental data, both in single and multiaxial loading (Budday et al., 2017; Comley and Fleck, 2012; Destrade et al., 2015; Franceschini et al., 2006; Mihai et al., 2015; Miller and Chinzei, 1997; 2002; Moran et al., 2014; Nicolle et al., 2004; Prange and Margulies, 2002; Rashid et al., 2012; 2013; 2014), and the relative errors from their nonlinear least-squares fit to the experimental data suggest that the models with higher order terms are more successful in approximating the data than the ones with lower order terms. However, even though Ogden-type constitutive models are widely appealing since they are readily implemented in many popular software packages, the fact that a relatively large number of parameters may be required to approximate the data to the desired accuracy makes them less attractive to users.

Here, our objective is to build a family of isotropic hyperelastic strain-energy functions, with a small number of parameters, that exhibit the characteristic behavior under combined shear and compression or tension. To achieve this, we devise a systematic strategy to derive such models and demonstrate their performance on experimental data for human brain tissue from Budday et al. (2017). Our algorithmic approach is generic and, as such, applicable to other biological tissues with similar properties, including adipose tissue. We start our analysis in Section 2, with a detailed study of the deformation of a cuboid of isotropic hyperelastic material under simple shear superposed on finite axial stretch. This analysis reveals the crucial role of the nonlinear shear modulus defined as the ratio between the shear stress and the shear strain. Unlike the linear shear modulus which is a constant, in general, the nonlinear shear modulus is a function of the deformation that enables us to identify characteristic behaviors under large shear superposed on large compression or tension. In Section 3, we show that, for a given shear superposed on finite compression or tension, the elastic behavior under large shear is consistent with that under small shear. This observation is used to identify a generic strain-energy function capable of predicting the physical behavior of human brain tissues subjected to combined shear and tension or compression. In Sections 4 and 5, we exploit our key observations from the experimental data and employ the nonlinear shear modulus to derive a family of hyperelastic models with a small number of parameters that predict the elastic behavior of brain tissue under combined shear and axial loading and are suitable for finite-element analyses.

2. Finite shear superposed on axial stretch

The finite elastic deformation of a material body is described by the mapping $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X})$, which defines a one-to-one correspondence between the positions of material points in the reference configuration $\mathbf{X} = (X, Y, Z) \in \mathcal{B}_0$ and their positions in the current configuration $\mathbf{x} = (x, y, z) \in \mathcal{B}$. Kinematics quantities for the deformation (Beatty, 1996; Truesdell and Noll, 2004) are associated with the deformation gradient

$$\mathbf{F} = \operatorname{Grad} \boldsymbol{\chi}. \tag{1}$$

Here, we restrict our attention to hyperelastic isotropic materials. These materials are described by a strain-energy function $\mathcal{W}(\lambda_1, \lambda_2, \lambda_3)$, where $\{\lambda_i\}_{i=1,2,3}$ are the principal stretches (Ogden, 1997, p. 94). We assume that the material is incompressible so that $J = \det \mathbf{F} = \lambda_1\lambda_2\lambda_3 = 1$. Then, the Cauchy stress tensor has the representation:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \beta_1\mathbf{B} + \beta_{-1}\mathbf{B}^{-1}, \tag{2}$$