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Interaction of rate- and size-effect using a dislocation density based strain gradient viscoplasticity model

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ABSTRACT

Size effects occur in non-uniform plastically deformed metals confined in a volume on the scale of micrometer or sub-micrometer. Such problems have been well studied using strain gradient rate-independent plasticity theories. Yet, plasticity theories describing the time-dependent behavior of metals in the presence of size effects are presently limited, and there is no consensus about how the size effects vary with strain rates or whether there is an interaction between them. This paper introduces a constitutive model which enables the analysis of complex load scenarios, including loading rate sensitivity, creep, relaxation and interactions thereof under the consideration of plastic strain gradient effects. A strain gradient viscoplasticity constitutive model based on the Kocks–Mecking theory of dislocation evolution, namely the strain gradient Kocks–Mecking (SG-KM) model, is established and allows one to capture both rate and size effects, and their interaction. A formulation of the model in the finite element analysis framework is derived. Numerical examples are presented. In a special virtual creep test with the presence of plastic strain gradients, creep rates are found to diminish with the specimen size, and are also found to depend on the loading rate in an initial ramp loading step. Stress relaxation in a solid medium containing cylindrical microvoids is predicted to increase with decreasing void radius and strain rate in a prior ramp loading step.

1. Introduction

Experiments where a non-uniform plastic deformation is present show that metals appear as more resistant to deformation when the structural dimensions are reduced to micrometer size and below. Bending (Stölken and Evans, 1998) and torsion of micrometer sized specimens (Fleck et al., 1994; Liu et al., 2013; Liu et al., 2012) indicate a dependence of the effective response on specimen size. Indentation experiments exhibit indentation size effect, i.e. a strengthening effect with reduced depth of indentation, (Feng and Nix, 2004; Poole et al., 1996; Swadener et al., 2002). Size-dependent deformation properties are found in fine-grained metal matrix composites (Tian et al., 2014) and multiphase steel (Lyu et al., 2015), where a decrease reinforcement particles diameter leads to an increase flow resistance (Legarth, 2015; Lloyd, 1994). Similarly, the flow stress in geometrically similar perforated plate under tension was found to increase as the void size was decreased

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Nomenclature

Quantity	Significance
b	magnitude of Burgers vector
$\mathbf{C}$	material tangent stiffness
$\mathbf{D}$	elasticity tensor
E	Young's modulus
k_1, k_2, k_{20}	parameters describing dislocation evolution law
K	bulk modulus
m, n, p	strain rate exponents
M	Taylor factor
$\bar{r}$	Nye factor
α	empirical parameter accounting for dislocation interactions
$\boldsymbol{\varepsilon}$	strain tensor
$\bar{\varepsilon}^{vp}$	equivalent viscoplastic strain
$\boldsymbol{\varepsilon}^{vp}$	viscoplastic strain tensor
$\dot{\bar{\varepsilon}}^{vp}$	equivalent viscoplastic strain rate
$\dot{\boldsymbol{\varepsilon}}^{e}$	elastic strain rate tensor
$\dot{\boldsymbol{\varepsilon}}^{vp}$	viscoplastic strain rate tensor
$\dot{\varepsilon}_0$	reference strain rate
ϕ	viscoplastic potential
$\bar{\eta}^{vp}$	effective strain gradient
μ, λ	Lamé's coefficients
Λ	mean free path of dislocation
ρ	average total dislocation density
ρ_0	initial value of the total dislocation density
ρ_S	statistically stored dislocations (SSDs) density
ρ_G	geometrically necessary dislocations (GNDs) density
$\rho_G^{\max}$	maximum allowable GNDs density
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\bar{\sigma}$	equivalent stress
σ_0	initial yield stress
σ_{flow}	flow stress
$\sigma_{\text{flow, s}}$	saturation value of flow stress
σ_{ref}	reference stress

(Taylor et al., 2002). Classical plasticity models are not able to explain such findings, but constitutive theories incorporating plastic strain gradients are able to capture such response.

The size effects in plasticity are an outcome of strong plastic strain gradients as introduced either by the load configuration or the geometry of the boundary value problem. Then, plastic deformation is related to two populations of dislocations. Statistically stored dislocations (SSDs) are associated with uniform plastic deformation while geometrically necessary dislocations (GNDs) are introduced for lattice compatibility in non-uniform plastic deformation (Ashby, 1970). In a small dimension controlled inhomogeneous plastic deformation case, the resulting deformation fields are associated with significant strain gradients and thus substantive GNDs densities. This in turn influences the local hardening behavior of the elasto-plastic materials.

Metals can exhibit rate-dependent deformation response. It is therefore natural to extend the question of the size-dependency to plastic deformation conditions where loading rates and hold times are considered. The contribution of time-dependent plastic flow at high temperature or rapid loading at room temperature in conjunction with the presence of strain gradients could be significant. Experimental evidence of combined rate and size effects are scattered, but important conclusions can be drawn. Measurements of indentation hardness reported in Franke et al. (2010) showed reduced indentation size effects with increasing temperature. In Li and Ngan (2004) strong size effects on the stress exponent at constant indentation loads were found. In Phani and Oliver (2016) the interplay between the size effects and viscoplastic behavior was investigated in nanoindentation creep experiments with constant rate and rate jump experiments. These experiments showed that the size effects are less pronounced at higher temperature. This finding was attributed to a higher mobility and increased dislocation density with increased temperature, also (Franke et al., 2010). Although Phani and Oliver (2016) considers only two rate values, a difference in the size effects was nevertheless observed. Other experimental studies also revealed a strong hold time dependence of the creep strain rate and strain-rate sensitivity on loading rate and peak load in load-control nanoindentation experiments (Han et al., 2010; Peykov et al., 2012). The varying influence of indentation size effects on strain-rate sensitivity measurements in indentation-creep experiments was also reported in Alkorta et al. (2008). Accord-