Journal of the Mechanics and Physics of Solids 102 (2017) 277–293

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Wrinkle-free design of thin membrane structures using stress-based topology optimization

Yangjun Luo, Jian Xing, Yanzhuang Niu, Ming Li*, Zhan Kang*

State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China

ARTICLE INFO

Article history:
Received 13 July 2016
Revised 21 January 2017
Accepted 5 February 2017
Available online 8 February 2017

Keywords:
Thin membrane
Wrinkle
Topology optimization
Principal stress criterion
Stress relaxation

ABSTRACT

Thin membrane structures would experience wrinkling due to local buckling deformation when compressive stresses are induced in some regions. Using the *stress criterion* for membranes in wrinkled and taut states, this paper proposed a new stress-based topology optimization methodology to seek the optimal wrinkle-free design of *macro-scale* thin membrane structures under stretching. Based on the continuum model and linearly elastic assumption in the taut state, the optimization problem is defined as to maximize the structural stiffness under membrane area and principal stress constraints. In order to make the problem computationally tractable, the stress constraints are reformulated into equivalent ones and relaxed by a cosine-type relaxation scheme. The reformulated optimization problem is solved by a standard gradient-based algorithm with the adjoint-variable sensitivity analysis. Several examples with post-bulking simulations and experimental tests are given to demonstrate the effectiveness of the proposed optimization model for eliminating stress-related wrinkles in the novel design of thin membrane structures.

1. Introduction

With the advantages of light weight, large deformation, ease of stowing and folding, thin membranes have been widely used in various fields, such as aerospace, communication and micro-electronics (Jenkins and Leonard, 1991; Jenkins, 1996). Typical applications include inflatable reflectors, radar antennas, solar sails, sun shields and stretchable electronics, etc. However, due to their very poor resistance of compression, thin membranes tend to wrinkle when external loads are imposed under certain boundary conditions.

Over the past decades, numerical simulation of the wrinkling behavior of membranes has attracted considerable interests. Earlier efforts on the analysis of wrinkling can be traced back to the 1930s, when Wagner (1929) and Reissner (1938) established a tension field (TF) theory. Since then, many mathematical models and numerical methods have been developed and applied to a wide variety of complicated membrane wrinkling problems. By appropriately changing the Poisson's ratio in wrinkled regions, Stein and Hedgepeth (1961) proposed a nonlinear wrinkle model to measure partially wrinkled membranes. Further, Ding and Yang (2003) extended Stein-Hedgepeth model to a new membrane model with variable Young's modulus and Poisson's ratio. Pipkin (1995) defined a relaxed energy function model for static analysis of wrinkling in membranes, and Epstein and Forcinito (2001) extended this model to wrinkling of anisotropic elastic membranes by introducing a general theory of saturated elasticity. By employing the geometrically nonlinear formulation and updated Lagrangian

* Corresponding authors.
E-mail addresses: mingli@dlut.edu.cn (M. Li), zhankang@dlut.edu.cn (Z. Kang).

http://dx.doi.org/10.1016/j.jmps.2017.02.003
0022-5096/

method, Su et al. (2003), Lee and Youn (2006), and Tessler et al. (2005) simulated the post-buckling behavior of thin-film membranes exhibiting wrinkling. In addition, finite-element dynamic analysis of wrinkled membranes undergoing large overall motion has been studied by e.g. Kang and Im (1999), Liu et al. (2013), and Miyazaki (2006). Moreover, the wrinkling behavior has been accurately predicted and analyzed in a lot of macroscopic and microscopic multidisciplinary problems by using the FEM simulation (Kang and Im, 1997; Senda et al., 2015; Cheney et al., 2015; Brau et al., 2013; Huang et al., 2014; Huang et al., 2015) or the multi-scale modeling (Xu and Potier-Ferry, 2016).

In most practical situations, wrinkling is undesirable because it will seriously undermine the performance and the reliability of membrane structures. For example, the occurrence of wrinkles could lead to a curved reflecting surface in membrane reflectors, or non-uniform surface heating in solar sails. How to effectively eliminate wrinkles in membranes has become a challenging problem. In order to reduce overall weight and mitigate wrinkling, Sakamoto et al. (2003) introduced a so-called cable suspended structure that consists of spider web-like cable layers next to the membrane boundary. Yan et al. (2014) tried to empirically control the wrinkle pattern by introducing simple microstructures such as circular holes into the membrane.

Design optimization for reducing wrinkles has also been extensively studied in the literature. Typically, Kim and Lee (2002) and Punurai et al. (2012) utilized the modified cutting pattern procedure and the genetic algorithm, respectively, to find the optimum cutting pattern by minimizing discrepancies between actual stress and design stress, and thus reduce the occurrence of wrinkling. Akita and Natori (2008) defined the wrinkle-reduction design as a wrinkle intensity-minimization problem, which was solved by using a semi-analytical sensitivity analysis method. Lim et al. (2012) presented a shape optimization approach to minimize the amplitude of wrinkles for membrane structures with reinforcement corner patches. Recently, Duan et al. (2015) developed a membrane shape-control model for the optimal design of the electrostatic forming membrane reflector to remove wrinkling and achieve a high membrane reflector precision.

As the literature survey reveals, most existing studies on the wrinkle-reduction of thin membrane structures focus on empirical designs or parameter optimization methods. However, during these design processes, the topological configuration or layout of the membrane is predefined and chosen purely by the intuition or experience of the designer. Thus, the need is highlighted to determine the optimal topology of membrane structures for efficiently eliminating wrinkling. To this end, topology optimization, which aims to find the best material distribution in a design domain, can be used as a promising tool.

Topology optimization has been extensively investigated since the pioneering work of Bendsøe and Kikuchi in 1988. Three main types of topology optimization methods (material density-based method, level set method and evolutionary structural optimization) have become popular and successfully applied to a wide range of design problems. The interested readers are referred to the review articles by Sigmund and Maute (2013) and Deaton and Grandhi (2014). In particular, much effort (Querin et al., 2010; Luo and Kang, 2012; Bruggi and Duysinx, 2013; Cai et al., 2016; Shi et al., 2016) has been devoted to the topology optimization problem with tension-compression asymmetry material or bi-modulus material, e.g. concrete, masonry and polymer. However, unlike the considered materials in existing studies, a thin membrane is characterized by zero bending stiffness and zero resistance of compression, and thus stretch-induced wrinkles might occur due to the Poisson's effect. Up to now, a topology optimization formulation has not yet been developed to generate optimal topologies for eliminating such wrinkles in membrane material, partly because of the difficulty in constructing an applicable optimization model and the complicated post-bulking behaviors exhibited by wrinkled membranes.

In this study, we focus on the wrinkle-free topology design of *macro-scale* thin membrane structures. The topology optimization problem is formulated as a new stress-based optimization model capable of generating optimal solutions that are less likely to form wrinkles. Based on the density-related model for material properties and the stress criterion in the evaluation of wrinkled/slack states, the considered optimization model is to maximize the structural stiffness under the constraints on total membrane area and principal stresses. An equivalent transfer function for principal stress constraints is employed to improve the computational efficiency. Meanwhile, a cosine-type relaxation scheme is developed to circumvent the stress singularity phenomenon and to avoid an overly narrow feasible domain in the design space. Then the method of moving asymptotes (MMA) (Svanberg, 1987) is adopted to update the design variables. Finally, topology optimization solutions of three numerical examples are provided to show the effectiveness of the proposed optimization method in novel wrinkle-free topology design of thin membranes.

2. Principal stress criterion for wrinkling

Essentially, wrinkles in thin membranes are local post-buckling patterns that are exhibited by large out-of-plane deformations. It is usually assumed that thin membranes have negligible bending stiffness and cannot resist compression stress. When a local in-plane compression state tends to occur within the membrane structure, the membrane will intermediately deflect and hence keep the in-plane principal stresses non-negative. Therefore, the mechanical behavior of the membrane can be divided into two distinct stages:

(a) **Before wrinkling**

In this stage, the membrane is still flat and can be described in a conventional state of plane stress. Here, we consider a two-dimensional membrane structure composed of isotropic elastic materials. Before occurrence of wrinkling, the membrane deformation is assumed to be small, and the equilibrium equation of the membrane with zero body forces in an x-y