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Indentation of a stretched elastomer

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ABSTRACT

Indentation has been intensively used to characterize mechanical properties of soft materials such as elastomers, gels, and soft biological tissues. In most indentation measurements, residual stress or stretch which can be commonly found in soft materials is ignored. In this article, we aim to quantitatively understand the effects of prestretches of an elastomer on its indentation measurement. Based on surface Green's function, we analytically derive the relationship between indentation force and indentation depth for a prestretched Neo-Hookean solid with a flat-ended cylindrical indenter as well as a spherical indenter. In addition, for a non-equal biaxially stretched elastomer, we obtain the equation determining the eccentricity of the elliptical contacting area between a spherical indenter and the elastomer. Our results clearly demonstrate that the effects of prestretches of an elastomer on its indentation measurement can be significant. To validate our analytical results, we further conduct correspondent finite element simulations of indentation of prestretched elastomers. The numerical results agree well with our analytical predictions.

1. Introduction

Indentation test has been intensively explored to measure mechanical properties of diverse materials ranging from metals (Corcoran et al., 1997; Ma and Clarke, 1995; Nix and Gao, 1998), ceramics (Gong et al., 1999; Pharr, 1998) to elastomers (Briscoe et al., 1998; VanLandingham et al., 2001) and biological tissues (Ebenstein and Pruitt, 2006; Oyen, 2010; Rho et al., 1997). In the experiment, indentation force is measured as a function of indentation depth, which can be used to assess fundamental mechanical properties of testing materials, such as elastic modulus and yield strength. Compared to many other mechanical testing methods including simple extension and compression tests, indentation test requires less or even no sample preparation, much smaller sample size and causes much less damage to the testing samples. In addition, by reducing the size of indenter, local mechanical properties of a heterogeneous material can be measured with high spatial resolution.

Recently, interest has been growing to use indentation test to characterize mechanical properties of gels and soft biological tissues (Constantinides et al., 2008; Ebenstein and Pruitt, 2004; Hu et al., 2012; Hu et al., 2010; Oyen, 2008). One salient feature of biological tissues and gels is that they are usually not in stress-free state. Large residual stress and strain can be commonly found in soft biological tissues (Fung, 1991; Fung and Liu, 1989; Rausch and Kuhl, 2013) as well as gels (Beebe et al., 2000; Hong et al., 2009). The residual stress and strain can be either isotropic or highly anisotropic (Amar and Goriely, 2005; Goriely et al., 2016; Marcombe et al., 2010). As a consequence, residual stress and strain may also exist in

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the sample for indentation test, though they can be partially or fully released through careful sample preparation. In particular, indentation tests have been recently explored to be directly conducted onto biological tissues without any sample preparation (Huang et al., 2005; Silver-Thorn, 1999; Vannah and Childress, 1996) to maintain their integrity and functionality. Therefore, measured relationship between indentation depth and indentation force should generally depend on both the mechanical properties and residual stress or strain state of testing samples. For instance, Zamir and Taber have shown that the calculated elastic modulus of soft biological tissues based on indentation test could be one order of magnitude higher than its true value if residual stress is ignored (Zamir and Taber, 2004). However, it is unclear how the residual stress or strain may quantitatively affect the indentation tests on soft materials.

According to our knowledge, Green, Rivlin, and Shield were the first who studied indentation of incompressible prestretched elastomers (Green et al., 1952). Later, the analysis was extended to compressible elastomers (Beatty and Usmani, 1975) and indenter with arbitrarily axisymmetric shape (Dhaliwal and Singh, 1978). Nevertheless, all the work was focused on the elastomer subject to equal-biaxial stretch, which greatly limits the application of their results. Filippova first studied indentation of a prestretched elastomer with non-equal biaxial prestretches (Filippova, 1978). To obtain analytical results of eccentricity of the contacting area between a spherical indenter and the elastomer as well as the indentation force as a function of indentation depth, the difference of two principle prestretches was assumed to be small. To study the effect of prestretch of an elastomer on its adhesion, Gay assumed deformation of elastomer stays in linear regime and took the prestretch into consideration by "mapping" the spherical indenter into an ellipsoidal one (Gay, 2000). However, the intuitive assumption has not been verified and may introduce significant errors.

Although indentation is generally a nonlinear problem, deformation in the material caused by indentation is typically small. Linear elasticity model was commonly adopted for the material in formulating indentation problem. In this article, we study indentation of an elastomer in a homogenously prestretched state. The prestretch in the elastomer can be arbitrarily large. However, following the commonly adopted assumption, additional deformation caused by indentation is assumed to be small. As a result, the problem can be regarded as a small deformation superimposed onto a finite deformation in a material. Following the method developed by Biot and Ogden (Biot, 1965; Ogden, 1997), we formulate the indentation of a prestretched elastomer with both flat-ended cylindrical indenter and spherical indenter in Section 2. To validate our analytical results, in Section 3, we conduct corresponding finite element simulations and compare the numerical results to our analytical predictions. Concluding remarks of our investigation are included in Section 4.

2. Analytical modeling

2.1. Surface Green's function for a biaxially stretched elastomer

Surface Green's function for a biaxially stretched elastomer occupying half space has been derived by Filippova (1978) and He (2008), in which the elastomer is taken to be a homogeneous, isotropic, and incompressible Neo-Hookean solid. Since we will use the derived surface Green's function to solve the current indentation problem in the article, we will briefly summarize the results obtained by Filippova and He in the following.

We first introduce a Cartesian coordinate (x_1, x_2, x_3) into the elastomer, which is bounded by the surface $x_3 = 0$, as shown in Fig. 1(a). The elastomer is under prestretch λ_1 and λ_2 along x_1 and x_2 directions. When a unit external force in x_j direction is applied onto the surface of the elastomer at $\boldsymbol{\xi} = (\xi_1, \xi_2, 0)$, the induced displacement in x_i direction at $\mathbf{x} = (x_1, x_2, 0)$ is defined as surface Green's function $\boldsymbol{G}_{ij}(\mathbf{x} - \boldsymbol{\xi})(i, j = 1, 2, 3)$. In the scenario that the additional strains induced by external surface forces are small, the incremental displacement field $u_i(\mathbf{x})$ in the elastomer resulted from any distributed loading can be obtained by linear superposition

In our following analysis, the contact between indenter and the elastomer is assumed to be frictionless. Therefore, surface Green's function component $\boldsymbol{G}_{33}(\mathbf{x} - \boldsymbol{\xi})$ is the only relevant one as given by Filippova (1978) and He (2008),

$$G_{33}\left(\mathbf{x} - \boldsymbol{\xi}\right) = \frac{\lambda_1^2 \lambda_2^2 (t + r) r}{2\pi \mu \left(t^3 + t^2 r + 3tr^2 - r^3\right)}, \tag{1}$$

in which μ is shear modulus of the elastomer, and r and t are defined by

$$r = \sqrt{(x_1 - \xi_1)^2 + (x_2 - \xi_2)^2}, \tag{2}$$

$$t = \lambda_1 \lambda_2 \sqrt{\lambda_1^2 (x_2 - \xi_2)^2 + \lambda_2^2 (x_1 - \xi_1)^2}. \tag{3}$$

The normal surface displacement $u_3(\mathbf{x})$ caused by a general normal pressure distribution $p(\mathbf{x})$ could be determined through the integration:

$$u_3(\mathbf{x}) = \iint_S G_{33}\left(\mathbf{x} - \boldsymbol{\xi}\right) p\left(\boldsymbol{\xi}\right) d\xi_1 d\xi_2, \tag{4}$$

where S is the area with nonzero pressure.