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How a dissimilar-chain system is splitting: Quasi-static, subsonic and supersonic regimes

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ABSTRACT

We consider parallel splitting of a strip composed of *two different chains*. As a waveguide, the dissimilar-chain structure radically differs from the well-studied identical-chain system. It is characterized by three speeds of the long waves, c_1 and c_2 for the separate chains, and $c_+ = \sqrt{(c_1^2 + c_2^2)/2}$ for the intact strip where the chains are connected. Accordingly, there exist three ranges, the subsonic for both chains $(0, c_2)$ (we assume that $c_2 < c_1$), the intersonic (c_2, c_+) and the supersonic, (c_+, c_1). The speed in the latter range is supersonic for the intact strip and at the same time, it is subsonic for the separate higher-speed chain. This fact allows the splitting wave to propagate in the strip supersonically.

We derive steady-state analytical solutions and find that the splitting can propagate steadily only in two of these speed ranges, the subsonic and the supersonic, whereas the intersonic regime is forbidden. In the case of considerable difference in the chain stiffness, the lowest dynamic threshold corresponds to the supersonic regime. The peculiarity of the supersonic mode is that the supersonic energy delivery channel, being initially absent, is opening with the moving splitting point.

Based on the discrete and related continuous models we find which regime can be implemented depending on the structure parameters and loading conditions. The analysis allows us to represent the characteristics of such processes and to demonstrate strengths and weaknesses of different formulations, quasi-static, dynamic, discrete or continuous.

Analytical solutions for steady-state regimes are obtained and analyzed in detail. We find the force-speed relations and show the difference between the static and dynamic thresholds. The parameters and energy of waves radiated by the propagating splitting are determined. We calculate the strain distribution ahead of the transition point and check whether the steady-state solutions are admissible.

1. Introduction

Fracture mechanics (and the theory of transitions under driving forces in a broader sense), founded by Griffith (1920) and Eshelby (1951, 1956), developed over decades in the framework of continuum mechanics. Novozhilov (1969a, 1969b), apparently for the first time, noticed the role of the medium discreteness and associated instabilities in the fracture process. Thomson et al. (1971) independently drew attention to the importance of these factors, while examining quasi-static split-

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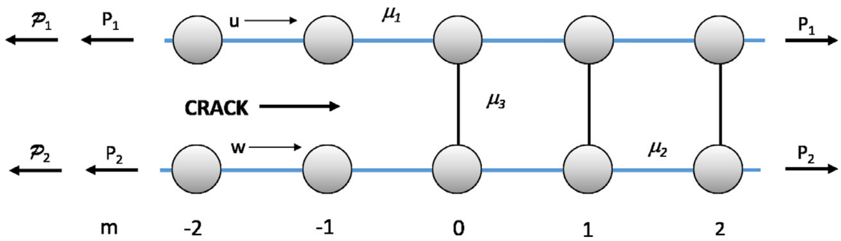

Fig. 1. The double-chain strip under the splitting. The initial and additional forces denoted by $P_{1,\,2}$ and $\mathcal{P}_{1,2}$, respectively, are applied far from the splitting point. The strains are initially the same, so $\mu_2 P_1 = \mu_1 P_2$.

ting of a double-chain strip. In their paper, the energy transferred to lattice oscillations the process of splitting was estimated and the term “lattice trapping” was introduced. In the first analytical solution for the crack dynamics in a two-dimensional lattice (Slepyan, 1981), the local-to-global energy release ratio was presented as a function of the crack speed. Thereby, the speed-dependent wave radiation energy was determined. We also refer to the books by Kunin (1975, 1982, 1983) on the microstructure in elasticity.

To date, analytical examination of the lattice fracture and phase transition is substantially developed. Numerical methods and results in the molecular dynamics of lattices are described, e.g., by Liu et al. (2006), Buehler (2008), and Buehler and Gao (2006). A comprehensive paper on the interface fracture mechanics is presented by Banks-Sills (2015). Concerning chains in biology see, e.g., a book by Alberts et al. (2002).

The pioneering work by Thomson et al. (1971) is the first among the publications most related to the problem under consideration. Slepyan and Troyankina (1984) studied a transition wave in a bistable chain. The dynamic splitting of a strip of two identical chains was considered analytically and numerically in Marder and Gross (1995) (the splitting under a constant load). Mishuris et al. (2009), Mishuris and Slepyan (2014), Ayzenberg-Stepanenko et al. (2014) and Slepyan et al. (2010) studied the splitting under a sinusoidal wave and in the presence of internal energy. The dynamical extraction of a single chain from a discrete lattice was examined in Mishuris et al. (2008). Lastly, the formulation and some results for the mode III subsonic splitting of a double-chain strip are presented in Mishuris et al. (2012).

While in the above works, *identical-chain systems* were considered, we now examine the quasi-static and dynamic splitting of a *dissimilar-chain system*. We note that similar splitting modes and phenomena can manifest themselves in both the molecular chains in biology and in macro-level composite structures. Dissimilarity changes the dynamic model dramatically. Compared with the well-studied identical-chain system (which has only a single wave speed), the considered structure represents a more complex waveguide having different physicalÂ properties. It is characterized by three wave speeds, c_1 and c_2 for the separate chains and c_+ for the system where the chains are connected. Accordingly, we will distinguish three wave ranges, the subsonic, $0 < v < c_2 < c_1$, intersonic, $c_2 < v < c_+$, and supersonic, $c_+ < v < c_1$. Note that the latter is subsonic for the higher speed chain. We can it the *supersonic splitting*, since the speed $v > c_+$ is greater than the wave speed in the intact strip.

We find that the splitting can propagate only in two of these speed ranges, the subsonic and the supersonic, whereas the steady splitting in the intersonic regime is impossible. The analytical solutions for both permitted regimes are obtained. We find the dependence of the splitting speed on the applied force and determine the quasi-static and sub- and supersonic dynamic thresholds. Also, the local-to-global speed-dependent energy release ratios defining the energy of radiated waves are presented.

Along with the chain structure in the quasi-static and dynamic regimes, we consider the corresponding continuous model. The results following from the continuum approximation can be of interest by themselves. They also present the global framework used in the analysis of the discrete system.

The quasi-static solution for the discrete model shows the role of the strip parameters (the difference in stiffness of the chains and the strength of the chain connecting links) and gives us the total energy release at low speeds (which corresponds to the energy of the radiated waves).

The steady-state solution is inadmissible if the splitting criterion is first achieved not at the splitting point, as assumed in the formulation, but ahead of it Marder and Gross (1995). Note that in this case, more complicated established regimes can form. The clustering and forerunning modes revealed in Mishuris et al. (2009) and Slepyan et al. (2015), respectively, are the examples.

Based on the obtained analytical results, we calculate the distribution of the chain-connecting-bond strain over the intact part of the strip, which allows us to check the admissibility.

In Conclusions, novel results presented in this paper are summarized.

2. The strip and the basic equations

The strip under consideration consists of two mass-spring chains connected by elastic bonds, Fig. 1. One of the chains (the upper one on the Figure) is marked by number 1 as well as its parameters. Consequently, number 2 is assigned