Journal of the Mechanics and Physics of Solids 98 (2017) 106–125

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Localization in inelastic rate dependent shearing deformations

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ARTICLE INFO

Article history:
Received 15 May 2016
Received in revised form
1 July 2016
Accepted 25 August 2016
Available online 17 September 2016

Keywords:
Shear bands
Hadamard instability
Turing instability
Shear localization
Strain-rate sensitivity

ABSTRACT

Metals deformed at high strain rates can exhibit failure through formation of shear bands, a phenomenon often attributed to Hadamard instability and localization of the strain into an emerging coherent structure. We verify formation of shear bands for a nonlinear model exhibiting strain softening and strain rate sensitivity. The effects of strain softening and strain rate sensitivity are first assessed by linearized analysis, indicating that the combined effect leads to Turing instability. For the nonlinear model a class of self-similar solutions is constructed, that depicts a coherent localizing structure and the formation of a shear band. This solution is associated to a heteroclinic orbit of a dynamical system. The orbit is constructed numerically and yields explicit shear localizing solutions.

1. Introduction

Shear bands are narrow zones of intense shear observed during the dynamic deformation of many metals at high strain rates. Shear localization forms a striking instance of material instability, often preceding rupture, and its study has attracted considerable attention in the mechanics (e.g. Clifton et al., 1984; Fressengeas and Molinari, 1987; Molinari and Clifton, 1987; Anand et al., 1987; Wright and Walter, 1987; Clifton, 1990; Tzavaras, 1992; Burns and Davies, 2002; Wright, 2002), numerical (e.g. Walter, 1992; Chen and Batra, 1999; Estep et al., 2001; Baxevanis et al., 2010), or mathematical literature (e.g. Dafermos and Hsiao, 1983; Tzavaras, 1986; Bertsch et al., 1991; Estep et al., 2001; Katsaounis and Tzavaras, 2009). In experimental investigations of high strain-rate deformations of steels, observations of shear bands are typically associated with strain softening response – past a critical strain – of the measured stress–strain curve (Clifton et al., 1984). It was proposed by Zener and Hollomon (1944), and further precise by Clifton et al. (1984) and Shawki and Clifton (1989), that the effect of the deformation speed is twofold: An increase in the deformation speed changes the deformation conditions from isothermal to nearly adiabatic, and the combined effect of thermal softening and strain hardening of metals may produce a net softening response. On the other hand, strain-rate hardening has an effect *per se*, inducing momentum diffusion and playing a stabilizing role.

Strain softening has a destabilizing effect; it is known at the level of simple models (e.g. Wu and Freund, 1984) to induce *Hadamard instability* – an ill-posedness of a linearized problem. It should however be remarked that, while Hadamard

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http://dx.doi.org/10.1016/j.jmps.2016.08.015
0022-5096/

instability indicates the catastrophic growth of oscillations around a mean state, what is observed in localization is the orderly albeit extremely fast development of coherent structures, the shear bands. Despite considerable attention to the problem of localization, little is known about the initial formation of shear bands, due to the dominance of nonlinear effects from the early instances of localization.

Our aim is to study the onset of localization for a simple model, lying at the core of various theories for shear band formation,

$$\partial_t v = \partial_x(\varphi(\gamma) v_x^n),$$
$$\partial_t \gamma = \partial_x v, \tag{1}$$

which will serve to assess the effects of strain softening $\varphi'(\gamma) < 0$ and strain-rate sensitivity $0 < n \ll 1$ and analyze the emergence of a shear band out of the competition of Hadamard instability and strain-rate hardening. The model (1) describes shear deformations of a viscoplastic material in the xy-plane, with v the velocity in the y-direction, γ the plastic shear strain (elastic effects are neglected), and the material is obeying a viscoplastic constitutive law of power law type,

$$\sigma = \frac{1}{\gamma^m}(\gamma_t)^n, \quad \text{corresponding to } \varphi(\gamma) = \gamma^{-m} \text{ with } m > 0. \tag{2}$$

The constitutive law (2) can be thought as describing a plastic flow rule on the yield surface. The model (1) captures the bare essentials of the localization mechanism proposed in Zener and Hollomon (1944), Clifton et al. (1984), and Shawki and Clifton (1989). Early studies of (1) appear in Hutchinson and Neale (1977) (in connection to necking), Wu and Freund (1984) (for linear rate-sensitivity) and Tzavaras (1986, 1991, 1992).

The uniform shearing solutions,

$$v_s(x) = x, \quad \gamma_s(t) = t + \gamma_0, \quad \sigma_s(t) = \varphi(t + \gamma_0), \tag{3}$$

form a universal class of solutions to (1) for any $n \geq 0$. When $n=0$ and $\varphi'(\gamma) < 0$ the system (1) is elliptic in the t-direction and the initial value problem for the associated linearized equation presents Hadamard instability; nevertheless it admits (3) as a special solution. Rate sensitivity $n > 0$ offers a regularizing mechanism, and the associated system (1) belongs to the class of hyperbolic–parabolic systems (e.g. Tzavaras, 1991). The linearized stability analysis of (3) has been studied in Fressengeas and Molinari (1987), Molinari and Clifton (1987), and Shawki and Clifton (1989) for (even more complicated models including) (1) and (2). As (3) is time dependent, the problem of linearized stability leads to the study of non-autonomous linearized problems. This was addressed by Fressengeas and Molinari (1987) and Molinari and Clifton (1987) who introduced the study of relative perturbations, namely to assess the stability of the ratios of the perturbation relative to the base time-dependent solution, and provided linearized stability results. Such linearized stability and instability results compared well with studies of nonlinear stability (Tzavaras, 1986 or Tzavaras, 1992 for a survey).

Here, we restrict to the constitutive function $\varphi(\gamma) = \frac{1}{\gamma}$ in (2) but retain the dependence in n, and study

$$v_t = \left(\frac{v_x^n}{\gamma}\right)_x, \quad \gamma_t = v_x. \tag{4}$$

This model has a special and quite appealing property: After considering a transformation to relative perturbations and a rescaling of variables,

$$v_x(x, t) =: u(x, t) = U(x, \tau(t)), \quad \gamma(x, t) = \gamma_s(t)\, \Gamma(x, \tau(t)), \quad \sigma(x, t) = \sigma_s(t)\, \Sigma(x, \tau(t)) \quad \text{where } \tau(t) = \log\left(1 + \frac{t}{\gamma_0}\right), \tag{5}$$

the problem of stability of the time-dependent uniform shearing solution (3) is transformed into the problem of stability of the equilibrium $(\bar{U}, \bar{\Gamma}) = (1, 1)$ for the nonlinear but *autonomous* parabolic system

$$U_\tau = \Sigma_{xx} = \left(\frac{U^n}{\Gamma}\right)_{xx}, \quad \Gamma_\tau = U - \Gamma. \tag{6}$$

The following heuristic argument leads to a conjecture regarding the effect of rate sensitivity n on the dynamics: As time proceeds the second equation in (6), which is of relaxation type, relaxes to the equilibrium manifold $\{U = \Gamma\}$. Accordingly, the stability of (6) is determined by the equation describing the effective equation

$$U_\tau = \left(U^{n-1}\right)_{xx}.$$

The latter is parabolic for $n > 1$ and backward parabolic for $n < 1$, what suggests instability in the range $n < 1$. The argument is proposed in Katsaounis and Tzavaras (2009) in connection to the development of an asymptotic criterion for the quantitative assessment of shear band formation and can be quantified by means of an asymptotic expansion (see Section 4).

In this paper we study the dynamics of the system (6). First, we provide a complete analysis of linearized stability. The linearized system around the equilibrium $(1, 1)$ takes the form