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Effect of Tabor parameter on hysteresis losses during adhesive contact

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ABSTRACT

The Tabor parameter μ is conventionally assumed to determine the range of applicability of the classical 'JKR' solution for adhesive elastic contact of a sphere and a plane, with the variation of the contact area and approach with load, and in particular the maximum tensile force (the pull-off force) being well predicted for $\mu > 5$. Here we show that the hysteretic energy loss during a contact separation cycle is significantly overestimated by the JKR theory, even at quite large values of μ. This stems from the absence of long-range tensile forces in the JKR theory, which implies that jump into contact is delayed until the separation $\alpha = 0$. We develop an approximate solution based on the use of Wu's solution with van der Waals interactions for jump-in, and the JKR theory for jump out of contact, and show that for $\mu > 5$, the predicted hysteresis loss is then close to that found by direct numerical solutions using the Lennard-Jones force law. We also show how the same method can be adapted to allow for contact between bodies with finite support stiffness.

1. Introduction

When adhesive forces [e.g. van der Waals forces] are included in the formulation of an elastic contact problem, the normal loading and unloading curves can be different, implying hysteresis losses during a loading/unloading cycle. This effect is most pronounced in small scale contacting systems such as the atomic force microscope [AFM]. In particular, if the AFM is operated in the tapping mode, hysteretic losses will be reflected in a phase shift between the oscillation of the tip and the driving excitation (Stark et al., 2001; Noy et al., 1998; Tamayo and Garcia, 1996; Rutzel et al., 2003).

Many analytical treatments of adhesive contact are based on the 'JKR' model (Johnson et al., 1971), in which tensile tractions are allowed within the contact area. The extent of this is then determined by minimizing the total energy, composed of elastic strain energy, interface energy and potential energy of external forces. Maugis and Barquins (1978) demonstrated that his procedure is exactly analogous to the formulation of linear elastic fracture mechanics [LEFM], with the energy release rate here being equated to the interface energy $\Delta\gamma$. This implies a unique value of the mode I stress intensity factor

$$K_I = -\sqrt{2\pi}\lim_{s\to 0} p(s)\sqrt{s} = -\sqrt{2E^*\Delta\gamma}, \tag{1}$$

where $p(s)$ is the contact pressure at a distance s from the boundary of the contact area and E^* is the composite contact modulus.

Tabor (1977) argued that the assumptions of the JKR theory are appropriate for contact between a sphere of radius R and a plane

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when the ‘Tabor parameter’

$$\mu = \frac{1}{\varepsilon}\left(\frac{R(\Delta\gamma)^2}{E^{*\,2}}\right)^{1/3} \tag{2}$$

is sufficiently large, where ε is a length characterizing the range of interaction of the adhesive forces. Subsequently, Derjaguin and his collaborators (Muller et al., 1980) showed by a major computing project how the contact behaviour varied from DMT (quasi-rigid) behaviour to JKR behaviour as the Tabor parameter increases, and in particular, that the maximum possible tensile force (the ‘pull-off force’) is predicted within a few percent by the JKR theory if $\mu > 5$. However, in this paper we shall show that the JKR theory significantly overestimates hysteresis loss, even at quite high values of μ.

2. The JKR equations

The JKR solution is obtained by taking a Hertzian contact between a sphere and a half space and then raising the sphere by a distance Δ without changing the contact radius a or disturbing the flatness: a flat punch problem. This gives the traction distribution

$$\sigma(r) = -\frac{2E^*\sqrt{a^2-r^2}}{\pi R} + \frac{E^*\Delta}{\pi\sqrt{a^2-r^2}}. \tag{3}$$

The requirement that the stress-intensity factor at the contact edge be given by Eq. (1) establishes the lift Δ as $\sqrt{2\pi a\Delta\gamma/E^*}$, so that the relative motion of distant points in the two bodies is

$$\alpha = \sqrt{\frac{2\pi a\Delta\gamma}{E^*}} - \frac{a^2}{R}. \tag{4}$$

Notice that α is here defined from the position where two equivalent rigid bodies just made point contact.[1] Integrating the tractions (3) and using the above value of Δ, we obtain the (tensile positive) force

$$T = \sqrt{8\pi E^* a^3\Delta\gamma} - \frac{4E^*a^3}{3R}. \tag{5}$$

Maugis (2000) has shown how the number of independent parameters in the JKR equations can be reduced. Following him, but omitting his numerical factors, we define the dimensionless variables

$$\hat{T} = \frac{T}{R\Delta\gamma};\quad \hat{\alpha} = \frac{\beta^2\alpha}{R};\quad \hat{a} = \frac{\beta a}{R}, \tag{6}$$

where β is a dimensionless parameter defined by

$$\beta = \left(\frac{E^*R}{\Delta\gamma}\right)^{1/3} = \sqrt{\frac{R}{\mu\varepsilon}}. \tag{7}$$

With this notation, the force–displacement relation is defined implicitly by the relations

$$\hat{\alpha} = \sqrt{2\pi\hat{a}} - \hat{a}^2;\quad \hat{T} = \sqrt{8\pi\hat{a}^3} - \frac{4\hat{a}^3}{3}, \tag{8}$$

and is plotted in Fig. 1.

We can also define a dimensionless measure of the work W done by the force T as

$$\widehat{W} \equiv \frac{W}{\mu\varepsilon R\Delta\gamma} = \int\hat{T}d\hat{\alpha} = \int\hat{T}\frac{d\hat{\alpha}}{d\hat{a}}d\hat{a} = \frac{8\hat{a}^5}{15} + \pi\hat{a}^2 - \frac{4\sqrt{2\pi\hat{a}^7}}{3}, \tag{9}$$

where the final expression is obtained by substituting for $\hat{T}$, $\hat{\alpha}$ from Eqs. (8) and evaluating the resulting integral. The dimensionless work done in traversing any finite segment of the curve in Fig. 1 can be obtained by imposing appropriate limits on this indefinite integral.

Consider a scenario in which the bodies are initially widely separated, but are caused to approach each other and subsequently separate under displacement control [rigid grips]. The approach phase must follow the positive $\hat{\alpha}$-semi-axis as far as the origin, since in the JKR solution no tractions are implied between surfaces with a positive local separation. However, further approach implies an unstable jump into contact at the point B in Fig. 1. The force–displacement relation then remains on this branch of the curve until an unstable jump out of contact occurs at point C during the separation phase. We note that when the load is varied during contact (e.g. moving back and forth along the solid line in Fig. 1), there is no hysteretic energy loss.

The parameter values at B and C can be determined from Eqs. (8) and the conditions

[1] For studying adhesive contact, it seems convenient to abandon the traditional contact mechanics convention where compressive loads and indentations are taken to be positive. Thus here, α is not an indentation, but a gap or separation.