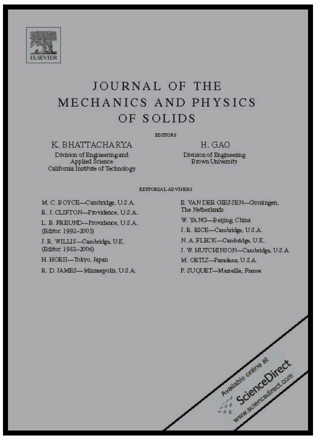

www.elsevier.com/locate/jmps

Instabilities of soft films on compliant substrates

M.A. Holland, B. Li, X.Q. Feng, E. Kuhl

PII: S0022-5096(16)30412-4
DOI: http://dx.doi.org/10.1016/j.jmps.2016.09.012
Reference: MPS2991

To appear in: *Journal of the Mechanics and Physics of Solids*

Received date: 17 June 2016
Revised date: 8 August 2016
Accepted date: 14 September 2016

Cite this article as: M.A. Holland, B. Li, X.Q. Feng and E. Kuhl, Instabilities o soft films on compliant substrates, *Journal of the Mechanics and Physics c Solids,* http://dx.doi.org/10.1016/j.jmps.2016.09.012

Instabilities of soft films on compliant substrates

M. A. Holland[a], B. Li[b], X. Q. Feng[b], E. Kuhl[a,c,*]

[a]*Department of Mechanical Engineering, Stanford University, Stanford, CA 94305, USA*
[b]*Institute of Biomechanics and Medical Engineering,Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China*
[c]*Department of Bioengineering, Stanford University, Stanford, CA 94305, USA*

Abstract

Instabilities in bilayered systems can generate a wide variety of patterns ranging from simple folds, wrinkles, and creases to complex checkerboards, hexagons, and herringbones. Physics-based theories traditionally model these systems as a thin film on a thick substrate under confined compression and assume that the film is orders of magnitude stiffer than the substrate. However, instability phenomena in soft films on soft substrates remain insufficiently understood. Here we show that soft bilayered systems are highly sensitive to the stiffness ratio, boundary conditions, and mode of compression. In a systematic analysis over a wide range of stiffness ratios, from $0.1 < \beta < 1000$, for eight different compression modes including whole-domain compression, substrate prestretch, and film growth, we observe significantly different instability characteristics in the low-stiffness-contrast regime, for $\beta < 10$. While systems with inverse stiffness ratios under whole-domain compression are unstable for a wide range of wrinkling modes, under film-only compression, the same systems display distinct wrinkling modes. Strikingly, these discrepancies disappear when using measures of effective strain, effective stiffness, and effective wavelength. Our study suggests that future instability studies should use these effective measure to standardize their findings. Our results have important applications in soft matter and living matter physics, where stiffness contrasts are low and small environmental changes can have large effects on morphogenesis, pattern selection, and the evolution of shape.

Keywords: instabilities, bifurcation, bilayered system, soft matter, living matter

---

1. Motivation

Bilayered systems, consisting of a thin film of one material on a thicker substrate of another, are found in a variety of applications [42]. Under compression, they can experience one of several instability modes, including wrinkling [18, 11], folding [49, 50], creasing [17, 29, 36], and cusping [51, 52]. In some instances, these instabilities are undesired failure modes [29]; in others, they are mechanical features that can be tuned for a desired property or performance [34]. Traditionally, bilayered systems for engineering applications involved a high stiffness ratio between the stiff top film and the the soft bottom substrate, $\beta = E_\mathrm{f}/E_\mathrm{s}$, as in sandwich panels [1] or silicon [35, 48] and gold [50] films on polydimethylsiloxane substrates. These systems were heavily studied, analytically [1, 7], experimentally [35, 36, 51], and numerically [33, 36, 52], as far back as the turn of the century. When researchers from the mechanics community became interested in complex biological folding patterns in layered soft tissues, they applied these findings to development [6, 11], morphogenesis [21, 26], and pattern formation [20, 41]. It was easy to assume, then, based on morphological similarities, that the stiffness ratio between the thin outer layer and the thick substrate was of the same order as these traditional systems. Thus early studies of brain development used stiffness ratios around $\beta = 10^1$ or 10^2 [44]. Yet, recent experiments have shown that, on the contrary, the stiffness ratio in ultrasoft tissues like the brain is close to or even less than unity [14, 53, 54].

Much less work has been done on the regime of stiff substrates with soft films – bilayered materials with 'inverse' stiffness ratios of $\beta < 1$. Recent studies have used the Föppl-von Kármán equations [22, 37] to study instabilities of ultrasoft materials [6, 11]. The Föppl-von Kármán equations describe the deflections of large thin plates, and are based on the fundamental assumption that the thickness of the plate does not change significantly during compression [21]. This assumption is not valid in the case of soft materials experiencing compressive strains up to 50%. One group has examined the bifurcation of the energy functional in the range of $0 < \beta < 5$ for an incompressible growing layer, or rather a prestretched substrate [52], while a few other groups have studied the instabilities of a homogenous system with $\beta = 1$ under compression [8, 17, 23]. An elegant way to model systems in the low-stiffness-contrast regime is to represent the structure as a multi-layered system with weak intermediate layers [38]. This approach seems particularly suitable for structures that are actually composed of multiple layers like the human cerebellum [39].

---

[*] Corresponding author

Email addresses: `mholla@stanford.edu` (M. A. Holland), `libome@tsinghua.edu.cn` (B. Li), `fengxq@tsinghua.edu.cn` (X. Q. Feng), `ekuhl@stanford.edu` (E. Kuhl)