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A unified phase-field theory for the mechanics of damage and quasi-brittle failure☆

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ABSTRACT

Being one of the most promising candidates for the modeling of localized failure in solids, so far the phase-field method has been applied only to brittle fracture with very few exceptions. In this work, a unified phase-field theory for the mechanics of damage and quasi-brittle failure is proposed within the framework of thermodynamics. Specifically, the crack phase-field and its gradient are introduced to regularize the sharp crack topology in a purely geometric context. The energy dissipation functional due to crack evolution and the stored energy functional of the bulk are characterized by a crack geometric function of polynomial type and an energetic degradation function of rational type, respectively. Standard arguments of thermodynamics then yield the macroscopic balance equation coupled with an extra evolution law of gradient type for the crack phase-field, governed by the aforesaid constitutive functions. The classical phase-field models for brittle fracture are recovered as particular examples. More importantly, the constitutive functions optimal for quasi-brittle failure are determined such that the proposed phase-field theory converges to a cohesive zone model for a vanishing length scale. Those general softening laws frequently adopted for quasi-brittle failure, e.g., linear, exponential, hyperbolic and Cornelissen et al. (1986) ones, etc., can be reproduced or fit with high precision. Except for the internal length scale, all the other model parameters can be determined from standard material properties (i.e., Young's modulus, failure strength, fracture energy and the target softening law). Some representative numerical examples are presented for the validation. It is found that both the internal length scale and the mesh size have little influences on the overall global responses, so long as the former can be well resolved by sufficiently fine mesh. In particular, for the benchmark tests of concrete the numerical results of load *versus* displacement curve and crack paths both agree well with the experimental data, showing validity of the proposed phase-field theory for the modeling of damage and quasi-brittle failure in solids.

1. Introduction

The mechanics of damage and failure in solids is of vital importance in predicting the limit load capacity and preventing the potential catastrophic collapse of engineering structures. Ever since the pioneering work of Ngo and Scordelis (1967) and Rashid (1968), various theoretical models, based on the plasticity theory, fracture and damage mechanics, etc., have been

☆ Dedicated to Prof. Jie Li on the occasion of his 60th birthday

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developed to characterize the material behavior with softening regimes. Computational approaches like the smeared and discrete crack methods have also been proposed.

However, great care should be taken when theoretical models and computational methods are used in the numerical or discrete context (e.g., the finite element method or FEM). For instance, owing to the loss of ellipticity and lack of internal length scale, the local material models for softening solids cannot be used straightforwardly. Otherwise, non-objective or erroneous results would be unavoidable, unless some extra strategies are introduced as in the nonlocal and gradient-enhanced models (Peerlings et al., 1996; Pijaudier-Cabot and Bažant, 1987). Similarly, the classical smeared and discrete crack methods both suffer from the issues of mesh-alignment dependence and spurious stress locking (Jirásek and Zimmermann, 1998; Rots, 1988). Even for the more advanced enriched FEM, e.g., the embedded and extended FEMs (Jirásek and Belytschko, 2002; Wu, 2011), etc., it is rather cumbersome to explicitly represent the crack topology and track the propagation path, particularly for crack branching and merging in 3-D cases.

Regarding the above facts, the phase-field method for fracture in solids has attracted extensive theoretical and computational investigations since the late 1990s; see Ambati et al. (2015) and the references therein. In this method, the so-called crack phase-field is introduced to characterize the smooth transition between the bulk and cracks. Propagating cracks are tracked automatically through evolution of the crack phase-field on a fixed background mesh. Consequently, complicate failure processes, e.g., initiation, propagation, nucleation, merging and branching of cracks in general 3-D situations can be elegantly treated in a unified approach with no need for extra *ad hoc* strategies. The tedious task of crack tracking is avoided, presenting a significant advantage over those discrete crack methods.

Broadly speaking, the phase-field method has been developed almost in parallel in the physics, mechanics and mathematics communities. The physics community (Aranson et al., 2000; Hakim and Karma, 2009; Karma and Kessler, 2001; Spatschek et al., 2011) adapts the phase transition approach originated by Ginzburg and Landau (Landau and Lifshitz, 1980) and generally develops dynamic models for brittle fracture. Contrariwise, the mechanics community (Amor et al., 2009; Kuhn and Müller, 2010) relies on the variational formulation of Griffith's theory for brittle fracture (Francfort and Marigo, 1998) and the regularized version later developed (Bourdin et al., 2000); more details can be found in Bourdin et al. (2008). The thermodynamic framework is also established (Miehe et al., 2010), with extension to dynamic fracture (Borden et al., 2014; Hofacker and Miehe, 2013). As important supplements, the applied mathematicians, e.g., Alicandro et al. (1999); Ambrosio et al. (2000); Braides et al. (1999), etc., establish the Γ-convergence of the regularized variational formulation to Griffith's theory as the length scale tends to zero. Interestingly, the phase-field formulation can also be regarded as a gradient damage model, provided the length scale is treated as a material property rather than a numerical parameter (Pham et al., 2011; Sicsic and Marigo, 2013). For brittle fracture the similarities and differences between the phase-field model and the implicit gradient-enhanced damage model (Peerlings et al., 1996) were recently clarified in de Borst and Verhoosel (2016), and the phase-field model is justified since it does not exhibit spurious unbounded damage growth of the localization band. In this work, we prefer to use the notation and methodology of the mechanics community and limit ourself to the quasi-static failure process.

Despite many noteworthy contributions, the phase-field modeling of localized failure in solids remains an open and challenging issue. In particular, the existing phase-field models apply dominantly to brittle fracture, with very few of them dealing with quasi-brittle failure. Verhoosel and de Borst (2013) proposed a phase-field model for cohesive fracture in which the crack path is known *a priori*; see also May et al. (2015); Vignollet et al. (2014). Recently, a phase-field model for cohesive fracture was proposed in Conti et al. (2015); Focardi and Iurlano (2017) with very simple prototype examples. However, cohesive zone models with general softening laws still cannot be considered. Therefore, it is pressing to develop a phase-field theory which can deal with general quasi-brittle failure, and if possible, the classical models for brittle fracture can also be recovered as its particular cases.

Aiming for the above goal, in this work we propose a unified phase-field theory for the mechanics of damage and quasi-brittle failure in solids within the framework of thermodynamics (Miehe et al., 2010). The crack phase-field and its gradient are introduced to regularize the sharp crack topology in a purely geometric context. Accordingly, the energy dissipation functional can be defined similarly to classical phase-field models for brittle fracture. The material behavior during the whole failure process is completely determined by two constitutive functions in terms of the crack phase-field. Generic expressions for these constitutive functions are postulated. In particular, besides those adopted in classical phase-field models for brittle fracture, novel constitutive functions optimal for quasi-brittle failure are determined in such a way that the proposed phase-field theory converges to a cohesive zone model as the internal length scale vanishes. Those general softening laws frequently adopted for quasi-brittle solids, e.g., linear, exponential, hyperbolic and Cornelissen et al. (1986) ones, etc., are reproduced or fit with high precision. Furthermore, except for the internal length scale, all the other model parameters are identified from standard material properties (i.e., Young's modulus, failure strength, fracture energy and the target softening law). Several representative numerical examples of benchmark tests are also presented for the validation.

Though the proposed unified phase-field method also starts from the regularized Griffith's theory, it owns some interesting characteristics compared to other similar ones. On the one hand, the energetic degradation function and crack geometric function postulated in existing phase-field models for brittle fracture do not apply to quasi-brittle failure any more. To solve this problem, general constitutive functions are first proposed, such that those popular phase-field models for brittle fracture, e.g., Miehe et al. (2010); Pham et al. (2011), etc., can be recovered as the particular examples. The constitutive functions optimal for quasi-brittle failure are then determined consistently in such way that the resulting mode-I behavior is equivalent to a well-defined cohesive zone model. Accordingly, the internal length scale does not affect the overall global responses