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# Mechanics of finite cracks in dissimilar anisotropic elastic media considering interfacial elasticity

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ABSTRACT

Interfacial crack fields and singularities in bimaterial interfaces (i.e., grain boundaries or dissimilar materials interfaces) are considered through a general formulation for two-dimensional (2-D) anisotropic elasticity while accounting for the interfacial structure by means of an interfacial elasticity paradigm. The interfacial elasticity formulation introduces boundary conditions that are effectively equivalent to those for a weakly bounded interface. This formalism considers the 2-D crack-tip elastic fields using complex variable techniques. While the consideration of the interfacial elasticity does not affect the order of the singularity, it modifies the oscillatory effects associated with problems involving interface cracks. Constructive or destructive "interferences" are directly affected by the interface structure and its elastic response. This general formulation provides an insight on the physical significance and the obvious coupling between the interface structure and the associated mechanical fields in the vicinity of the crack tip.

## 1. Introduction

Solutions to the asymptotic singular fields at an interface crack tip between two materials play a central role in many micromechanics and computational models that are used to study the fracture behavior of engineering materials containing dissimilar material interfaces. Historically, the peculiarity of the near-tip behavior for an interface crack has been studied in both the isotropic bimaterial case (Inglish, 1913; Williams, 1959; Rice, 1988; Shih and Asaro, 1990; Suo and Hutchinson, 1990; Gao, 1991) and the anisotropic case (Gotoh, 1967; Clements, 1971; Willis, 1971; Ting, 1986; Bassani and Qu, 1989a; Qu and Bassani, 1989; Suo, 1990; Wu, 1990; Ni and Nemat-Nasser, 1991; Gao et al., 1992) using a variety of mathematical techniques including the Green's function tensor (Gao, 1991), the Stroh formalism (Eshelby et al., 1953; Stroh, 1958, 1962; Deng, 1993), or a distribution of interfacial dislocations (Willis, 1971; Gautesen and Dundurs, 1988; Qu and Li, 1991). Several basic crack problems using these techniques have been solved with an emphasis on the near-tip oscillatory field and the definition of the stress intensity factors for the crack-tip singularity.

In a majority of the referenced works above, the interface is assumed to be perfectly bonded, i.e., those models assume the continuity of both the traction and displacement across the interface. Theoretical models looking into imperfect/weak interfaces have also been developed for both isotropic (Cheng et al., 1996; Antipov et al., 2001; Sudak, 2003) and anisotropic materials (Pan, 2003; Sudak and Wang, 2006; Vellender et al., 2016). These models consider spring-like boundary conditions for which the interfacial displacement jump is directly proportional to the traction. Such boundary conditions at the interface change the character of the crack-tip singularity significantly (Mishuris and Kuhn, 2001).

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More recently, due to the growing interests in the so-called field of nanomechanics, the above mentioned seminal developments for perfectly bonded interfaces have been augmented by incorporating surface elasticity effects on the crack surface (Feng et al., 2008; Antipov and Schiavone, 2011; Kim et al., 2011; Nan and Wang, 2012; Koguchi and Suzuki, 2014; Wang et al., 2015; Sigaeva and Schiavone, 2016). The inclusion of the Gurtin and Murdoch (1975) surface elasticity theory in these models has been shown to also modify the nature and functional form of the singularity at the crack-tip.

The questions associated with the nature of the fields near a crack-tip are of importance in the context of the Irwin fracture criterion (Irwin, 1957) and require a precise knowledge of the local conditions at the crack-tip. Thus, all of the studies mentioned above (with or without surface elasticity effects) have shown that, for perfectly bonded interfaces, the order of the singularity of a crack lying on the interface is affected by the elastic constants mismatch (c.f. Dundurs, 1969 parameters), while the oscillatory singularities are no more than a consequence of the linear elasticity theory. A number of approaches have been proposed to cure this ill-behaved elasticity solution. For special combinations of the elastic constants, such as two incompressible materials or one incompressible and the other rigid under plane strain (Comninou, 1990) or for specific mathematical assumptions (England, 1965; Comninou, 1977) on the boundary conditions (frictionless contact near the tip), the elastic fields near the crack-tip follow a square-root singularity with no oscillations. However the elastic mismatch between the elastic constants of perfectly bonded interfaces or artificial "non-perfect" conditions on the interface are a mere idealization of complex phenomena at the interface and does not completely describe the structural and chemical characteristics of the interface separating both media. The corollary of the above assertion is that any theoretical framework describing interface crack problems needs to not only include the mismatch between the elastic constants but also the interfacial structural mismatch accounting for the effects of the interface structure (both cracked and uncracked regions) and its associated elastic behavior.

In this manuscript, the two-dimensional (2-D) problem of an interface crack between two anisotropic elastic solids is considered. The interfacial structure and state of coherency at the interface is accounted for by means of an interfacial elasticity paradigm (Section 2). This interface model introduces boundary conditions that are effectively equivalent to those for a weakly bounded interfaces (Appendix B). Using these boundary conditions, the elastic fields (i.e., displacements and stresses) in the vicinity of a crack are derived based on the Stroh formalism (Section 3). A discussion on the order of the singularity and the oscillatory effects appearing in problems involving interface cracks concludes this manuscript (Section 4).

A list of notation and convention used throughout this manuscript is cataloged in Appendix A.

## 2. Basic equations of elasticity considering surface elasticity

Consider two dissimilar semi-infinite linear elastic anisotropic half spaces having stiffness elasticity tensors $\mathbb{C}^+$ and $\mathbb{C}^-$ and being separated by an interface $S$ (located at $x_2 = 0$). Medium "+" and medium "−" are defined for $x_2 > 0$ and $x_2 < 0$ respectively. For convenience and clarity, all the field variables associated with the problem addressed here are confined to the two-dimensional space $x_1$–$x_2$. If we denote the displacement, strain and stress fields by $\boldsymbol{u}$, $\boldsymbol{\epsilon}$, and $\boldsymbol{\sigma}$, respectively, the Hooke's law in general anisotropy reads,

$$\sigma_{ij} = \mathbb{C}_{ijkl}\epsilon_{kl}^{el}, \tag{1}$$

where Roman indices range from 1 to 3 and Greek indices are taken as either 1 or 3 (i.e. in the plane of the interface), unless otherwise indicated. A comma followed by an index indicates the partial differentiation with respect to the corresponding coordinate variable and repeated indices are summed.

This bimaterial is subjected to a homogeneous traction and displacement boundary condition at infinity, such that far away from the interface the deformation can be assumed homogeneous, and the transverse stress and the in-plane strain tensors in the upper ($S_+$) and lower ($S_-$) materials are given by $\boldsymbol{\sigma}^{\perp,\infty} = (\sigma_{21}^{\infty}, \sigma_{22}^{\infty}, \sigma_{23}^{\infty})$ and $\boldsymbol{\epsilon}^{S,\infty} = (\epsilon_{11}^{\infty}, \epsilon_{33}^{\infty}, 2\epsilon_{13}^{\infty})$ respectively. It has been shown that such homogeneous deformation can be easily constructed in a bimaterial (Qu and Bassani, 1993) by the so-called "T-decomposition".

Following Dingreville et al. (2014), the in-plane strain $\boldsymbol{\epsilon}^S$ assumes continuity of displacement along the interface i.e., $\boldsymbol{\epsilon}^S = \boldsymbol{\epsilon}_+^S = \boldsymbol{\epsilon}_-^S$. Additionally, as described in more detail in Section 3, a second strain measure in the form of a coherency eigenstrain $\boldsymbol{\epsilon}^{*,S}$ is considered. This strain measure is associated with the interface to represent the state of coherency between the two solids (Dingreville et al., 2014). It describes the relaxation of lattice mismatches resulting in the formation of peculiar interfacial structures. For a coherent interface, the mismatch is completely accommodated by straining both phases. In the case of a semi-coherent interface, localized misfit dislocations are assumed to be responsible for compensating uniform far-field elastic fields, while an incoherent interface is the result of two rigid semi-infinite media in rigid contact (Romanov et al., 1998; Romanov and Wagner, 2001).

### 2.1. Lekhnitskii-Eshelby-Stroh (LES) representation in 2-D anisotropic elasticity

Problems of two-dimensional anisotropic elasticity as described above can be treated by the so-called Lekhnitskii-Eshelby-Stroh (LES) formalism (Eshelby et al., 1953; Stroh, 1958; Lekhnitskii, 1963). This approach can be considered as an extension of the stress function used in isotropic elasticity for anisotropic problems. The Navier equation governing the elasticity solutions is given by,

$$\mathbb{C}^{\pm}:\nabla\cdot\nabla\boldsymbol{u}_{\pm}(\mathbf{x}) = \mathbb{C}^{\pm}:\nabla\boldsymbol{\epsilon}_{\pm}^{*,S}(\mathbf{x}), \tag{2}$$

where $\boldsymbol{u}_{\pm}(\mathbf{x})$ is the displacement field in medium "+" and "−" respectively. To avoid confusion and cumbersome notation, the subscripts ± are ignored in the remainder of this section. Leveraging the two-dimensionality of the problem, the Navier equation (2)