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Incomplete contacts in partial slip subject to varying normal and shear loading, and their representation by asymptotes

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ABSTRACT

We develop a method for the solution of partial slip contact problems suffering complex loading cycles where, generally, the normal load, shear force and, potentially, differential bulk tensions are all functions of time, using an edge-asymptote approach. The size of the slip zone and local shear traction distribution are revealed as functions of time. The results are then re-worked in asymptotic form, so that they do not hinge on inherent symmetry and anti-symmetry conditions for the contact overall, and are of general applicability. The multipliers on the local solutions (generalised stress intensity factors) are also appropriate as a means of taking laboratory tests quantifying fretting fatigue and employing them to wholly different prototypical problems.

1. Introduction

Fretting fatigue, the accelerated nucleation of cracks caused by small amounts of differential movement between components pressed together, is conveniently divided into two phases; the nucleation of cracks and their propagation. Here, a rigorous quantification of the edge slip conditions for incomplete (convex) contacts is sought. The results to be derived are applicable to any geometry of contact which is capable of local idealization using half-plane analysis, i.e. to almost any contact which advances as the normal load is increased; it is necessary neither to have a closed form solution for the contact, nor even for the contact as a whole to be capable of idealization by half plane (or space) theory. The results found can be used in conjunction with geometries where only a numerical method such at the finite element procedure may be used to solve the contact problem, and which may be used to abstract the generalised stress intensity factors required as inputs here. In this paper, however, they are applied to simple problems where closed form algebra is possible for comparison.

The first solution for partial slip in contact was by Cattaneo who studied the Hertz problem (Cattaneo, 1938). Mindlin independently found the same solution many years later (Mindlin, 1949) and went on to look at further loading paths (Mindlin et al., 1951; Mindlin and Deresiewicz, 1953). Around the same time, Galin (1945) and Galin and Gladwell (2008) solved the partial slip problem of a rigid flat punch indenting an elastic half-plane. The next development was the inclusion of the effects of differential tension (Nowell and Hills, 1987), but a big development came when, independently, Jäger (1997) and Ciavarella (1998) noted that, for any geometry of half-plane contact, the corrective shear traction in the stick region was a scaled form of the sliding distribution. This has proved useful in many partial slip solutions, and a simplified notation developed by Barber et al. (2011) enabled the problem of partial slip problems involving varying normal load to be attacked. Separately, partial slip edge solutions were developed from the Cattaneo solution but where the edge direct and shear tractions were captured using asymptotic forms (Dini and Hills,

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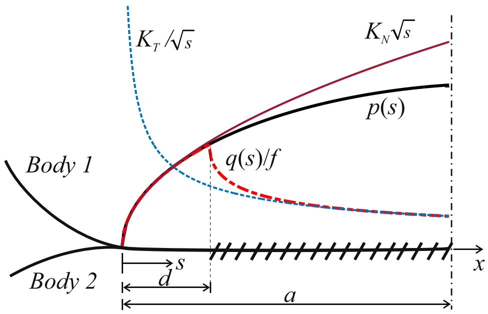

Fig. 1. Normal square root bounded asymptote and tangential square root singular asymptote.

2004; Dini et al., 2005). These followed the work of Sackfield et al. (2003) who first introduced asymptotic solutions to characterising the edge of contacts. This has the big advantages that they are (a) independent of the overall contact geometry, (b) remove the need for symmetry/anti-symmetry across the whole contact, and (c) as a consequence of (a) may be used as quantifiers of fretting fatigue enabling laboratory specimens test to be applied to very different problems (Hills and Dini, 2006; Hills et al., 2012). The Ciavarella-Jäger theorem cannot be used when the slip zones at each end of the contact have opposite signs. This restriction is removed if we concentrate our attention on one end only. One possibility explored here is to represent the behaviour of each end by a set of simple asymptotic forms.

The step made by Barber et al. (2011) was very significant and helpful in solving a number of real partial slip problems where the normal load varied, but these solutions suffered from the inherent limitation that the shear traction had to have its origin in the form of an applied shear force only whereas, in practice, differential bulk tensions in the bodies is equally important. The equivalence of the two methods of excitation for a constant normal load was discussed exhaustively by Hills et al. (2016a) for the case of a constant normal load, and here, we combine the concepts in Barber et al. (2011) and Hills et al. (2016a) to enable us to solve for edge slip in contacts where normal load, shear force and (potentially) bulk tension all vary in time. The method hinges on taking the results in Barber et al. (2011), attaching asymptotic forms to them, and noting that the origin of the shear stress intensity factor may be from either a shear force or differential tension.

Initially, we assume that an incomplete contact of half-width a is fully stuck, so that the local shearing traction distribution, $q(x)$, at the left hand edge of the contact, $x \to -a^+$, is given by

$$q(x) = \frac{K_T}{\sqrt{x+a}} = \frac{K_T}{\sqrt{s}}, \tag{1}$$

where $s = a + x$ (see Fig. 1) and the generalised stress intensity factor, K_T, may be induced by a shear force, Q, or a differential bulk tension, σ_0. The generalised stress intensity factor K_T can be excited by the shear traction and/or a differential bulk tension and is given by[1]

$$K_T = \frac{\pm Q}{\pi\sqrt{2a}} + \frac{\sigma_0}{2}\sqrt{\frac{a}{2}}. \tag{2}$$

Note that this result is wholly independent of the geometry of the problem in cases where the body may be thought of as a half-plane. Where the contact geometry is such that only local half-plane behaviour arises, such as a pin located in an almost conforming hole, more complicated definitions will be needed. The contact problem will be inherently uncoupled, and so we may write the local contact pressure at the end $x \to -a^+$ in the form

$$p(x) = K_N\sqrt{a+x} = K_N\sqrt{s}. \tag{3}$$

Here the value of the generalised stress 'intensity' factor is geometry dependent. Interestingly, the same information is incorporated into the instantaneous contact law, $a(P)$, and it may be shown (see Appendix A) that

$$K_N = \frac{1}{\pi}\sqrt{\frac{2}{a}}\frac{\mathrm{d}P}{\mathrm{d}a}, \tag{4}$$

where P is the normal load on the contact giving rise to a contact of half-width a. Note that only a limited number of problems will have a closed form contact law. For these an analytical equation for K_N may also be obtained. Example calibrations of K_N were summarised by Hills et al. (2016a). For more complex problems a semi-analytical formulation may be obtained by using the finite element method to find the calibration. We also record that the calibration of the problem of an elastic flat and rounded semi-infinite punch was obtained by Fleury et al. (2016a,2016b). The other pre-requisite for this paper is an understanding of the Ciavarella-Jäger theorem applied to a problem defined in terms of these asymptotic forms, and this is set out in Appendix B.

These edge asymptotes are illustrated in Fig. 1. Note, however, that there is one more possible form of asymptote needed, and which arises when both the shear and normal forces applied to the contact change, but $|\mathrm{d}Q/\mathrm{d}P| < f$. In this case the additional near-

[1] See Hills et al. (2016a) for a discussion of the sign conventions employed at each edge of the contact.