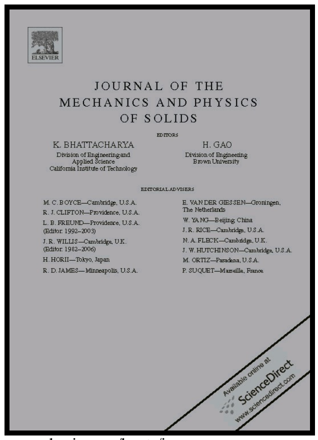

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Homogenization of a Cauchy continuum towards a micromorphic continuum

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Abstract

The micromorphic theory of Eringen and Mindlin, including special cases like strain gradient theory or Cosserat theory, is widely used to model size effects and localization phenomena. The heuristic construction of such theories based on thermodynamic considerations is well-established. However, the identification of corresponding constitutive laws and of the large number of respective constitutive parameters limits the practical application of such theories.

In the present contribution, a closed procedure for the homogenization of a Cauchy continuum at the microscale towards a fully micromorphic continuum is derived including explicit definitions of all involved generalized macroscopic stress and deformation measures. The boundary value problem to be solved on the microscale is formulated either for using static or kinematic boundary conditions. The procedure is demonstrated with an example.

Keywords: micromorphic theory; homogenization; gradient theory; generalized continua

1. Introduction

Classical theories of continuum mechanics can only be applied when the macroscopic wavelengths of relevant field quantities are much larger than the characteristic microstructural dimensions, a limitation manifested already in the lack of an intrinsic length scale in such continuum theories. However, in many engineering problems this condition is not fulfilled, e. g. for micro and nanodevices or if material instabilities lead to a localization of deformations.

In principle, generalized continuum theories can overcome these limitations. Among the generalized theories of continuum mechanics, micromorphic theory of Eringen and Mindlin [8, 9, 10, 33] has an outstanding role since it incorporates many others like the Cosserat theory or strain gradient theory as special cases. For a recent review and a comprehensive classification the reader is referred to [18, 30]. Phenomenologically, it is well-established how such theories can be constructed based on macroscopic thermodynamic considerations and/or the principle of virtual powers. Thereby, additional, generalized stresses occur which appear in additional balance equations [7, 9, 16, 19, 24]. This requires to formulate respective additional constitutive relations and to identify the corresponding constitutive parameters. Mostly, for this purpose classical constitutive laws are generalized heuristically by linear and reversible approaches for the generalized stress measures (i. e. quadratic ansatzes in the thermodynamic potentials) for simplicity, e. g. [7, 16]. In particular in extension of highly non-linear classical constitutive laws, this is questionable and leads to unrealistic predictions as pointed out recently in [17, 23].

Homogenization of the heterogeneous microstructure offers a solution to this problem. Regarding classical continuum mechanics, the homogenization procedure, whereby the macroscopic stresses and strains are defined as the volume averages of their microscopic counterparts and can be prescribed via corresponding boundary conditions at the microscale, is established already for decades [21, 22]. For the so-called couple stress theory (constrained Cosserat theory) the additional boundary conditions of bending-type were specified intuitively as well [1, 5, 13]. However, for the homogenization from a Cauchy