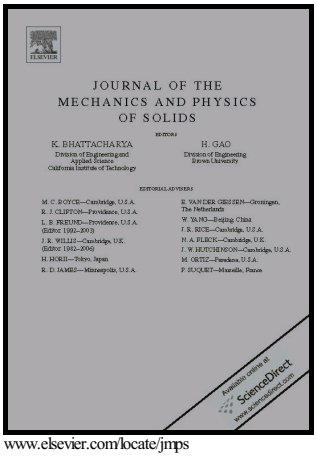
Negative refraction in a laminate

J.R. Willis



www.elsevier.com/locate/jmps

PII: S0022-5096(15)30262-3
DOI: http://dx.doi.org/10.1016/j.jmps.2015.11.004
Reference: MPS2743

To appear in: *Journal of the Mechanics and Physics of Solids*

Received date: 13 March 2015
Accepted date: 17 November 2015

Cite this article as: J.R. Willis, Negative refraction in a laminate, *Journal of th Mechanics and Physics of Solids,* http://dx.doi.org/10.1016/j.jmps.2015.11.004

Negative refraction in a laminate

J.R. Willis
Department of Applied Mathematics and Theoretical Physics
Centre for Mathematical Sciences
Wilberforce Road, Cambridge CB3 0WA, UK

November 18, 2015

Abstract

This work is concerned with the reflection and transmission of waves at a plane interface between a homogeneous elastic half-space and a half-space of elastic material that is periodically laminated. The lamination is always in the direction of the x_1-coordinate axis and the displacement is always longitudinal shear, so that the only non-zero displacement component is $u_3(x_1, x_2, t)$. After an initial discussion of Floquet–Bloch waves in the laminated material, brief consideration is given to the reflection-transmission problem, when the interface between the two media is the plane $x_1 = 0$. Nothing unusual emerges: there are just a single reflected wave and a single transmitted wave, undergoing positive group-velocity refraction. Then, the problem is considered when the interface between the two media is the plane $x_2 = 0$. The periodic structure of the interface induces an infinite set of reflected waves and an infinite set of transmitted waves. All need to be taken into account, but most decay exponentially away from the interface. It had previously been recognised that, if the incident wave had appropriate frequency and angle of incidence, a propagating transmitted wave would be generated that would undergo negative group-velocity refraction – behaviour usually associated with a metamaterial. It is established by example in this work that there is, in addition, a propagating transmitted wave with smaller wavelength but larger group velocity, that undergoes positive group-velocity refraction. The work concludes with a brief discussion of this finding, including its implications for the utility (or not) of "effective medium" theory.

Keywords: Laminated material, Floquet–Bloch waves, group velocity, refraction, metamaterial.