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On the determination of missing boundary data for solids with nonlinear material behaviors, using displacement fields measured on a part of their boundaries

Stéphane Andrieux [a,*], Thouraya N. Baranger [b]

[a] ONERA, Chemin de la Hunière, BP 80100, 92123 Palaiseau, France
[b] Université de Lyon, CNRS, LMC2 – Université Lyon1, 69622 Villeurbanne, France

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ABSTRACT

The paper is devoted to the derivation of a numerical method for expanding available mechanical fields (stress vector and displacements) on a part of the boundary of a solid into its interior and up to unreachable parts of its boundary (with possibly internal surfaces). This expansion enables various identification or inverse problems to be solved in mechanics. The method is based on the solution of a nonlinear elliptic Cauchy problem because the mechanical behavior of the solid is considered as nonlinear (hyperelastic or elastoplastic medium). Advantage is taken of the assumption of convexity of the potentials used for modeling the constitutive equation, encompassing previous work by the authors for linear elastic solids, in order to derive an appropriate error functional. Two illustrations are given in order to evaluate the overall efficiency of the proposed method within the framework of small strains and isothermal transformation.

1. Introduction

The problem of using overspecified measured boundary data on a part of a solid (displacement and stress vector fields) in order to extend the mechanical fields within the solid, or to identify missing or unknown boundary conditions, is still partially open despite potential applications being extremely numerous in mechanical and material sciences as well as in industry. Advances in the development of digital cameras and image correlation techniques (DIC) now make it possible to have measurement means for full field surface displacements that are cheap and easy to manage and, more importantly, leading to very large amounts of information (see for example, Avril et al., 2008; Sutton et al., 2009). This information obtained at the surface has been used in the literature to define a large number of inverse or identification problems with various applications (see Avril et al., 2008; Bonnet and Constantinescu, 2005; Grediac and Hild, 2013) and references therein. In the same spirit, Moireau et al. (2009) make use of alternative measurement techniques, such as Tagged Magnetic Resonance Imaging with the Harp algorithm (Osman et al., 2000). Nevertheless, the use of these surface data is still largely restricted either to qualitative estimation or to quantitative analysis based on a plane mechanical state or on homogeneous-through-the-thickness assumptions. The development of accurate and efficient methods for the 3D extension of the displacement field measured on a stress free surface would lead to a lot of new applications in mechanics. For instance, in analyzing complex mock-ups or experiments (3D numerical imaging,

* Corresponding author.
E-mail addresses: Stephane.Andrieux@onera.fr (S. Andrieux), Thouraya.Baranger@univ-lyon1.fr (T.N. Baranger).

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identification of boundary conditions in Andrieux and Baranger, 2008a) or physical parameters entering into their description, such as the friction coefficient, in identifying geometrical defects (Baranger and Andrieux, 2009) or contact surfaces (Andrieux and Baranger, 2012), in designing new non-destructive analysis or monitoring techniques, in computing linear fracture mechanics parameters (Andrieux and Baranger, 2013), etc.

One promising approach in dealing with this problem is to first reformulate it within the continuous framework, taking advantage of the fact that the amount and spatial density of the information obtained using the digital image correlation techniques makes it possible to consider that the complete displacement field is available on a part of the boundary and is not reduced on it to pinpoint data only, and second to then reformulate it as a Cauchy or Data Completion Problem, taking into account the fact that an overspecified data pair is given on a part of the boundary. Cauchy problems or Data Completion Problems belong to the class of inverse problems and are usually ill-posed in the sense of Hadamard (1923).

Linear elliptic Cauchy problems have been extensively studied since the 1920s (Hadamard, 1923) and compatibility conditions between the Cauchy data are known to be met, in order to ensure existence (Fursikov, 2000). Theoretical results for existence and data compatibility conditions for nonlinear elliptic Cauchy problems have been addressed by Klüger and Leitao (2003). Numerous numerical approaches are available in the literature for linear elliptic Cauchy problems, although the complexity of the algorithms and the large amount of computation needed limit the applications in almost all of the papers to two-dimensional problems: fixed point algorithms (Kozlov et al., 1992; Baumeister and Leitao, 2001; Marin and Lesnic, 2004); variational approaches based on Steklov–Poincaré operators (Ben Belgacem and El Fekih, 2005; Mejdi et al., 2006); least-squares method with vanishing regularization (Cimetière et al., 2001); methods using fundamental solutions (Marin and Lesnic, 2004; Young et al., 2008); or boundary integral techniques (Marin and Lesnic, 2002); moment methods associated with Backus–Gilbert techniques (Hon and Wei, 2001); quasi-reversibility methods (Bourgeois, 2005, 2006); and lastly energy error based methods (Andrieux and Baranger, 2008a, 2012, 2013; Andrieux et al., 2006; Baranger and Andrieux, 2008, 2009, 2011; Escriva et al., 2007) on the spirit of which this paper is based.

Very few papers can be found for nonlinear operators. Baumeister and Leitao (2001) proposed to solve a nonlinear Cauchy problem for a nonlinear scalar conduction equation by a change of variable and the solution of a linear Cauchy problem, but this method cannot be extended generally to other operators. The existence of a solution for a nonlinear Cauchy problem has also been studied by Egger and Leitão (2009) and Klüger and Leitao (2003), by a constructive method using a fixed point algorithm similar to the one designed by Kozlov et al. (1992).

In this paper, nonlinear solids are addressed and an extension of the variational method previously designed by the authors for linear elasticity is developed for convex hyperelastic solids and for dissipative solids governed by an elastoplastic constitutive relation described in the Generalized Standard Materials format (Halphen and Nguyen, 1975). For this last application, the derivation of the class of the error in a constitutive equation is based on previous work within the context of material parameter identification by Hadj-Sassi (2007), Hadj-Sassi and Andrieux (2006).

The following part is devoted to the definition of the Cauchy problem for nonlinear solids and the derivation of the variational methods. Parts 3 and 4 address the question of building an error that can ground the functional to be minimized in order to obtain the solution of the Data Completion problem, respectively for hyperelastic materials and elastoplastic ones.

2. Reformulation as a Cauchy problem

Let Ω be given a regular domain, the boundary of which is decomposed into three non-overlapping parts Γ_m, Γ_b, and Γ_u, for instance see Fig. 1. The usual boundary conditions are given on Γ_b (combination of normal stress vector and displacement vector components). Γ_m (the subscript m stands for measurements) is the part where, using DIC acquisition, both displacement and stress vector components (usually zero for the latter) are available, and make up an overspecified boundary data pair. Lastly, Γ_u is the remaining part of the boundary, where no boundary data is known:

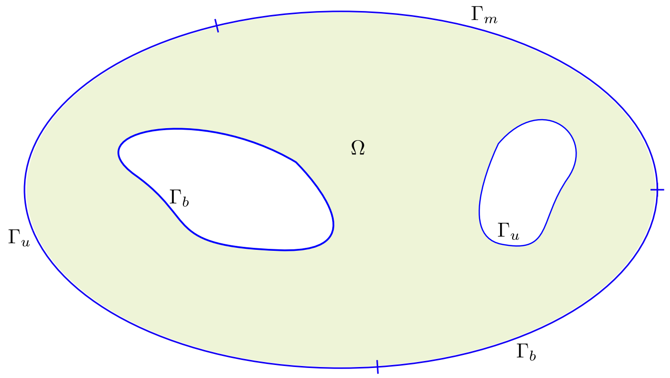

Fig. 1. The domain Ω and its boundaries.