ARTICLE IN PRESS

Journal of the Mechanics and Physics of Solids ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Localization of deformation and loss of macroscopic ellipticity in microstructured solids

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ARTICLE INFO

Article history:
Received 22 April 2016
Received in revised form
21 June 2016
Accepted 7 July 2016

Keywords:
Nonlinear Elasticity
Composites
Bifurcation
Instabilities
Homogenization
Cellular Solids

ABSTRACT

Localization of deformation, a precursor to failure in solids, is a crucial and hence widely studied problem in solid mechanics. The continuum modeling approach of this phenomenon studies conditions on the constitutive laws leading to the loss of ellipticity in the governing equations, a property that allows for discontinuous equilibrium solutions. Micro-mechanics models and nonlinear homogenization theories help us understand the origins of this behavior and it is thought that a loss of macroscopic (homogenized) ellipticity results in localized deformation patterns. Although this is the case in many engineering applications, it raises an interesting question: is there always a localized deformation pattern appearing in solids losing macroscopic ellipticity when loaded past their critical state?

In the interest of relative simplicity and analytical tractability, the present work answers this question in the restrictive framework of a layered, nonlinear (hyperelastic) solid in plane strain and more specifically under axial compression along the lamination direction. The key to the answer is found in the homogenized post-bifurcated solution of the problem, which for certain materials is supercritical (increasing force and displacement), leading to post-bifurcated equilibrium paths in these composites that show no localization of deformation for macroscopic strain well above the one corresponding to loss of ellipticity.

1. Introduction

Localization of deformation in finitely strained ductile solids is the instability mechanism leading to failure by rupture. The general principles were introduced for the study of this fascinating and important for applications phenomenon in the context of continuum mechanics by Hadamard (1903) and subsequently advanced in his spirit by Hill (1962), Mandel (1966) and Rice (1976). The underlying mathematical concept in the continuum model is the loss of ellipticity in the governing equations, which allows for discontinuous strain solutions. With the advent of homogenization theories since the 1960s,a vast amount of work has been dedicated to the bridging of scales and understanding how micromechanical features in

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http://dx.doi.org/10.1016/j.jmps.2016.07.009
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solids lead to their macroscopic (homogenized) loss of ellipticity at adequate levels of strain or stress. A plethora of applications for a wide range of solids has appeared in the literature, covering rubber elasticity, various types of composites (porous, fiber-reinforced, particle-reinforced, cellular solids, etc.), metal plasticity, granular media, rocks, just to name a few. Since the review of such a large and diverse body of work is unfortunately not possible, only key references relevant to the points made in the present article will be cited.

To avoid difficulties related to microstructure geometry and the identification of associated scale and representative volume, our attention is restricted to solids with a well defined scale, i.e. to architectured materials with periodic microsctructures. The role played by buckling at the microscopic scale, as the onset of instability mechanism leading to macroscopic localization of deformation in these materials has been established and subsequently analyzed by a long series of investigations. For the case of fiber reinforced composites, the connection between local buckling and global localization started with the work of Rosen (1965), who recognized microbuckling as the onset of instability mechanism. Subsequent investigations of Budiansky (1983), Budiansky and Fleck (1993), Kyriakides et al. (1995), Vogler et al. (2001) and many others showed, with progressively more sophisticated experiments and detailed modeling, how the buckling instability evolves into a localized deformation pattern (kinkband formation) and studied in detail the characteristics of these bands. The same basic mechanism, i.e. buckling initiated at the microstructural level, has been recognized in materials science as the cause for localization of deformation in cellular solids (crushing zones) and the interested reader is referred to the comprehensive monograph by Gibson and Ashby (1988). Detailed experimental and theoretical investigations followed in mechanics with a particular interest in studying the initiation and evolution towards localization of the deformation pattern in cellular solids by Papka and Kyriakides (1994, 1998, 1999a, 1999b) for 2D microstructures and Jang et al. (2010), Wilbert et al. (2011) for 3D microstructures and in establishing conditions where local or global buckling is the critical mechanism at the onset of failure by Triantafyllidis and Schraad (1998), Gong et al. (2005), and Lopez-Jimenez and Triantafyllidis (2013).

Progressing in parallel, the nonlinear homogenization theories that appeared in mechanics first addressed questions on macroscopic response in plasticity, viscoelasticity and nonlinear elasticity with various microstructures (e.g. see Suquet, 1983; Talbot and Willis, 1985; Ponte Castañeda, 1991) and subsequently explored localization of deformation issues (e.g. see Kailasam and Ponte Castañeda, 1998; Lopez-Pamies and Ponte Castañeda, 2004). For periodic solids the question asked was the possibility of detecting instabilities at the microscopic level from their homogenized properties, thus formally connecting buckling at the microscopic level to localization of deformation. For these composites it has been shown, initially for layered solids by Triantafyllidis and Maker (1985) and subsequently for the general 3D periodic case by Geymonat et al. (1993), that microstructural bifurcation phenomena (micro-buckling) is the mechanism responsible for macroscopic loss of ellipticity and that a long wavelength critical mode (based on Bloch wave analysis of the perfect infinite composite) coincides with the loss of ellipticity in its homogenized incremental moduli. Further work for porous elastomers by Michel et al. (2007) and for particle reinforced elastomers by Michel et al. (2010) has been done to connect local buckling to the macroscopic loss of ellipticity and compare periodic to random isotropic media with the same volume fractions.

Since loss of ellipticity is the property allowing for discontinuous equilibrium solutions, it is thought (and supported by micromechanical calculations in most of the known—to the best of our knowledge—engineering applications), that a loss of macroscopic (homogenized) ellipticity results in a localized deformation pattern in the post-bifurcated regime. However, an interesting question arises: is there always a localized deformation appearing in the post-bifurcation of solids losing macroscopic ellipticity and what are the necessary conditions in the homogenized response leading to localization?

In the interest of relative simplicity and analytical tractability, the present work answers these questions in the restrictive framework of an infinite, layered, nonlinear (hyperelastic) solid under plane strain loading conditions and more specifically under axial compression along the lamination direction. For this problem, based on a periodic unit cell construction, one can find macroscopic loads where the homogenized moduli of the principal solution lose ellipticity (and since the solid has an energy density, the corresponding homogenized energy loses rank-one convexity). Moreover one can also ensure that the critical (i.e. corresponding to the lowest applied load) bifurcation eigenmode of the infinite solid is global (infinite wavelength eigenmode), a property that for this problem allows us to find an homogenized solution for the post-bifurcated equilibrium path. The answer to the localization question posed lies in the homogenized, initial post-bifurcation response of the perfect layered solid, as seen in Fig. 1; it will be shown that for a composite with a monotonically increasing force (and

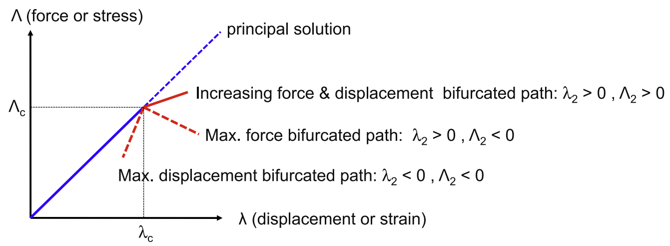

Fig. 1. Different cases for the homogenized, initial post-bifurcation behavior of a perfect, nonlinear (hyperelastic) layered composite under plane strain loading conditions which is subjected to axial compression along its lamination direction. Stable paths are marked by solid lines and unstable ones by broken lines. For a composite with a monotonically increasing force (and displacement) post-bifurcation response ($\lambda_2 > 0$, $\Lambda_2 > 0$), no localized deformation solution develops in spite of a loss of ellipticity found at the macroscopic critical strain λ_c.

Please cite this article as: Santisi d'Avila, M.P., et al., Localization of deformation and loss of macroscopic ellipticity in microstructured solids. J. Mech. Phys. Solids (2016), http://dx.doi.org/10.1016/j.jmps.2016.07.009