Journal of the Mechanics and Physics of Solids ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Estimates for the overall linear properties of pointwise heterogeneous solids with application to elasto-viscoplasticity

Martín I. Idiart [b,c], Noel Lahellec [a,*]

[a] Laboratoire de Mécanique et d'Acoustique, CNRS UPR 7051, Aix-Marseille University, Ecole Centrale Marseille, 4 impasse Nikola Tesla, 13453 Marseille Cedex 13, France

[b] Departamento de Aeronáutica, Facultad de Ingeniería, Universidad Nacional de La Plata, Avda. 1 esq. 47 S/N, La Plata B1900TAG, Argentina

[c] Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), CCT La Plata, Calle 8 No 1467, La Plata B1904CMC, Argentina

A R T I C L E I N F O

Article history:
Received 2 March 2015
Received in revised form
11 September 2015
Accepted 21 December 2015

Keywords:
Heterogeneous materials
Elasto-viscoplasticity
Variational methods
Homogenization

A B S T R A C T

New estimates are derived for the overall properties of linear solids with pointwise heterogeneous local properties. The derivation relies on the use of 'comparison solids' which, unlike comparison solids considered previously, are themselves pointwise heterogeneous. The estimates are then exploited within an incremental homogenization scheme to determine the overall response of multiphase elasto-viscoplastic solids under arbitrary loading histories. By way of example, the scheme is applied to incompressible Maxwellian solids with power-law plastic dissipation; particularly simple estimates of the Hashin–Shtrikman type are obtained. Predictions are confronted with full-field simulations for particulate composites under cyclic and rotating loading conditions. Good agreement is found for all cases considered. In particular, elasto-plastic transitions, tension-compression asymmetries (Bauschinger effect) and stress-path distortions induced by material heterogeneity are all well-captured, thus improving significantly on commonly used elastic-plastic decoupled schemes.

1. Scope

Many problems of practical interest in solid mechanics require knowledge of the overall properties of heterogeneous solids in terms of pointwise varying microscopic properties. Prominent examples include the role of residual stresses on the failure processes of engineering alloys and metal composites (e.g., Withers, 2007) and the effect of spontaneous electric polarization on the electromechanical response of ferroelectric composites (e.g., Miehe and Rosato, 2011; Idiart, 2014). Theoretical interest on pointwise heterogeneous solids has also arisen from recent developments initiated by Lahellec and Suquet (2003) on incremental homogenization schemes for predicting the macroscopic response and underlying field statistics in multiphase systems with elasto-viscoplastic behavior. These schemes rely on an implicit time discretization of the elasto-viscoplastic evolution equations together with a suitably designed variational principle governing the state of the solid at the end of the time step, given the state at the beginning of the time step. The latter enters into the scheme as pointwise heterogeneous stress polarizations. Incremental schemes of this sort are intended to improve on classical approaches based on the elasto-plastic tangent approximation (e.g., Hill, 1965) and the affine approximation (e.g., Masson

* Corresponding author.
E-mail addresses: martin.idiart@ing.unlp.edu.ar (M.I. Idiart), lahellec@lma.cnrs-mrs.fr (N. Lahellec).

http://dx.doi.org/10.1016/j.jmps.2015.12.017
0022-5096/

et al., 2000), as well as on decoupled approximations treating the elastic and plastic deformation processes separately (e.g., Aravas and Ponte Castañeda, 2004; Segurado et al., 2012). A recent account on the various schemes so far proposed can be found in Lahellec and Suquet (2013). In any event, central to these incremental schemes is the availability of accurate estimates for the overall properties of pointwise heterogeneous solids with linear local behavior.

It is precisely with a view to developing improved incremental homogenization schemes for elasto-viscoplastic systems that we focus here on linearly viscous solids with a local behavior characterized by

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{L}(\mathbf{x})\dot{\boldsymbol{\varepsilon}}(\mathbf{x}) + \boldsymbol{\tau}(\mathbf{x}), \tag{1}$$

where $\boldsymbol{\sigma}(\mathbf{x})$ and $\dot{\boldsymbol{\varepsilon}}(\mathbf{x})$ denote the Cauchy stress and the infinitesimal strain-rate tensor fields, while $\mathbf{L}(\mathbf{x})$ and $\boldsymbol{\tau}(\mathbf{x})$ denote the local viscosity and pre-stress or stress polarization tensors, respectively, both of which can vary with position $\mathbf{x}$ within the solid. The material response (1) can be written in terms of a quadratic potential function as

$$\boldsymbol{\sigma} = \frac{\partial w}{\partial \dot{\boldsymbol{\varepsilon}}}(\mathbf{x}, \dot{\boldsymbol{\varepsilon}}) \quad \text{where } w(\mathbf{x}, \dot{\boldsymbol{\varepsilon}}) = \frac{1}{2}\dot{\boldsymbol{\varepsilon}}\cdot\mathbf{L}(\mathbf{x})\dot{\boldsymbol{\varepsilon}} + \boldsymbol{\tau}(\mathbf{x})\cdot\dot{\boldsymbol{\varepsilon}}, \tag{2}$$

and, assuming that the viscosity tensor is everywhere positive-definite and that the microstructure exhibits separation of length scales, the overall response of the solid can be written in terms of an effective potential as (see, for instance, Ponte Castañeda and Suquet, 1998)

$$\overline{\boldsymbol{\sigma}} = \frac{\partial \widetilde{W}}{\partial \dot{\overline{\boldsymbol{\varepsilon}}}}(\dot{\overline{\boldsymbol{\varepsilon}}}) \quad \text{where } \widetilde{W}(\dot{\overline{\boldsymbol{\varepsilon}}}) = \min_{\dot{\boldsymbol{\varepsilon}}\in\mathcal{K}(\dot{\overline{\boldsymbol{\varepsilon}}})} \langle w(\mathbf{x}, \dot{\boldsymbol{\varepsilon}})\rangle. \tag{3}$$

In this expression, $\langle\cdot\rangle$ denotes volume averaging over a representative volume element Ω of the heterogeneous solid, $\overline{\boldsymbol{\sigma}} = \langle\boldsymbol{\sigma}\rangle$ and $\dot{\overline{\boldsymbol{\varepsilon}}} = \langle\dot{\boldsymbol{\varepsilon}}\rangle$, and $\mathcal{K}(\dot{\overline{\boldsymbol{\varepsilon}}})$ denotes the set of kinematically admissible strain-rate fields with prescribed volume average $\dot{\overline{\boldsymbol{\varepsilon}}}$.

A general procedure to estimate the effective response (3) in terms of low-order statistics of the local properties (2) has been recently proposed by Lahellec et al. (2011).[1] The procedure relies on the use of 'comparison solids' with *piecewise* heterogeneous properties whose effective potential can be easily estimated or computed exactly. A suitably designed variational statement is used to select the optimal properties that deliver the best possible estimate for the effective potential of the pointwise heterogeneous solid in terms of the effective potential of the piecewise-heterogeneous comparison solid. The resulting estimates have the virtue of improving, necessarily, on elementary estimates based on first-order statistics only, and of delivering bounds under certain conditions. It has been found, however, that the estimates can be quite inaccurate for some classes of pointwise heterogeneous solids which, incidentally, can be relevant to the development of incremental homogenization schemes. It is the case, for instance, of solids with strongly fluctuating polarization fields that are divergence-free. A remedy is proposed in Section 2 by allowing for comparison solids with *pointwise* heterogeneous properties in the variational procedure of Lahellec et al. (2011). This is achieved by making judicious use of divergence-free and compatible fields as comparison polarization stresses. The new linear estimates are then employed in Section 3 within a variant of the incremental homogenization scheme of Lahellec and Suquet (2013) to estimate the overall response of elasto-viscoplastic multiphase systems made up of Maxwellian phases. In the case of incompressible solids with power-law plastic dissipation, particularly simple estimates of the Hashin–Shtrikman type are obtained. Sample results for complex loading histories are confronted with decoupled estimates and full-field simulations in Section 4. Finally, some concluding remarks are given in Section 4.2.

2. Estimates based on pointwise heterogeneous comparison solids

2.1. Variational framework

We begin by introducing a 'comparison solid' with local potential

$$w_0(\mathbf{x}, \dot{\boldsymbol{\varepsilon}}) = \frac{1}{2}\dot{\boldsymbol{\varepsilon}}\cdot\mathbf{L}_0(\mathbf{x})\dot{\boldsymbol{\varepsilon}} + \boldsymbol{\tau}_0(\mathbf{x})\cdot\dot{\boldsymbol{\varepsilon}}, \tag{4}$$

where $\mathbf{L}_0(\mathbf{x})$ and $\boldsymbol{\tau}_0(\mathbf{x})$ are local properties within a certain class C to be specified. Then, upon defining the function

$$V(\mathbf{L}_0, \boldsymbol{\tau}_0) = \sup_{\dot{\boldsymbol{\varepsilon}}(\mathbf{x})} \left\langle w(\mathbf{x}, \dot{\boldsymbol{\varepsilon}}) - w_0(\mathbf{x}, \dot{\boldsymbol{\varepsilon}}) \right\rangle, \tag{5}$$

we have the inequality

$$\langle w(\mathbf{x}, \dot{\boldsymbol{\varepsilon}})\rangle \le \langle w_0(\mathbf{x}, \dot{\boldsymbol{\varepsilon}})\rangle + V(\mathbf{L}_0, \boldsymbol{\tau}_0) \tag{6}$$

for any set of admissible comparison properties and strain-rate field. In view of (3), this inequality generates the upper

[1] The procedure was proposed in the mathematically analogous context of thermoelasticity.

Please cite this article as: Idiart, M.I., Lahellec, N., Estimates for the overall linear properties of pointwise heterogeneous solids with application to elasto-viscoplasticity. J. Mech. Phys. Solids (2016), http://dx.doi.org/10.1016/j.jmps.2015.12.017