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Linear normal stress under a wheel in skid for wheeled mobile robots running on sandy terrain

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Abstract

Wheeled mobile robots are often used on high risk rough terrain. Sandy terrains are widely distributed and tough to traverse. To successfully deploy a robot in sandy environment, wheel-terrain interaction mechanics in skid should be considered. The normal and shear stress is the basis of wheel-soil interaction modeling, but the normal stress in the rear region on the contact surface is computed through symmetry in classical terramechanics equations. To calculate that directly, a new reference of wheel sinkage is proposed. Based on the new reference, both the wheel sinakge and the normal stress can be given using a quadratic equation as the function of wheel-soil contact angle. Moreover, the normal stress can be expressed as a linear function of the wheel sinkage by introducing a constant coefficient named as sand stiffness in this paper. The linearity is demonstrated by the experimental data obtained using two wheels and on two types of sands. The sand stiffness can be estimated with high accuracy and it decreases with the increase of skid ratio due to the skid-sinkage phenomenon, but increases with the increase of vertical load. Furthermore, the sand stiffness can be utilized directly to compare the stiffness of various sandy terrains.

Keywords: Terramechanics; Wheeled mobile robots; New reference of wheel sinkage; Linear normal stress; Sand stiffness

1. Introduction

Wheeled mobile robots are often used in such various areas as agriculture, defense and security, planetary surface exploration, and mining due to their robust mechanical structure and high maneuverability (Iagnemma et al., 2010; Yu et al., 2010). To successfully deploy a robot in an unstructured, outdoor environment, the interaction of the robot and terrain should be considered. Sandy terrains are widely distributed across terrestrial, lunar and Martian surfaces, and are difficult to traverse due to resulting wheel slip/skid and sinkage (Yang et al., 2014). Significant slip/skid and sinkage can result in reduced thrust and increased motion resistance and cause path tracking and localization errors.

Slip and skid terramechanics are two key components of the wheel-terrain interaction mechanics, and skid terramechanics for wheeled mobile robots is our research focus. A longitudinal skid model will be presented by the authors based on both Bekker's normal stress equation and the assumption that the maximum normal stress angle does not coincide with the angular position of the transition point of shear stress (Gao et al., 2013).

Integrating the distributed normal and shear stress under a wheel on deformable terrains to acquire the wheel-soil interaction force and torque is always utilized by scholars (Gao et al., 2013; Yoshida and Hamano, 2002; Wong and Reece, 1967a,b; Meirion-Griffith and

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Nomenclature

W	vertical load of wheel (N)
F_V	vertical force exerted on the wheel (N)
F_P	pushing force exerted on the wheel (N)
F_R	resistance force from the soil (N)
T_B	braking torque (N m)
K	shearing deformation modulus of soil (mm)
V_r	relative velocity of a point in the wheel rim tangential to the soil (mm/s)
K_σ	sand stiffness (Pa/m)
s_d	skid ratio
r	wheel radius (mm)
v	forward velocity of the wheel (mm/s)
p	pressure between plate and soil (Pa)
z	plate sinkage (mm)
z_1	wheel sinkage (mm)
z_2	rebounding height of soil (mm)
z_σ	wheel sinkage relative to the new reference (mm)
n	sinkage exponent of soil
k_c	cohesive modulus of soil (Pa/m^{n+1})
k_φ	frictional modulus of soil (Pa/m^n)
c	cohesion of soil (Pa)
j	shearing deformation distance (mm)
b	wheel width (mm)
ω	angular velocity of wheel (rad/s)
θ_1	entrance angle of wheel (°)
θ_2	leaving angle of wheel (°)
θ_m	angular position of the maximum radial stress (°)
θ_0	angular position of the transition point of tangential stress (°)
τ	tangential stress (Pa)
φ	internal friction angle of soil (°)
σ	radial stress (Pa)
σ_1	radial stress in the front region (Pa)
σ_2	radial stress in the rear region (Pa)
k_1, k_2, k_3	coefficients used for computing normal stress or wheel sinkage
k_{11}, k_{12}	coefficients used for calculating wheel sinakge
k_{21}, k_{22}	coefficients used for calculating normal stress
$f_1(\theta)$	function to calculate wheel sinkage
$f_2(\theta)$	function to calculate normal stress

Spenko, 2013, 2011; Ishigami et al., 2007a,b; Iagnemma et al., 2004; Iagnemma and Dubowsky, 2004; Ding et al., 2015; Nagatani et al., 2009; Wong and Huang, 2006; Irani et al., 2011). That is to say the distributed stress is the basis of wheel-soil interaction modeling. The first pressure-sinkage expression used in terramechanics was proposed in 1913 by Goriatchkin (Bernstein, 1913). The model was developed based on the fact that the pressure–sinkage relationship for a flat plate fits the form of a power function.

Then, Bekker established a pressure–sinkage model containing two deformation modules in 1955, which was validated by the experimental data obtained from a terrestrial vehicle locomotion laboratory (Bekker, 1969). Reece modified Bekker's model by introducing two dimensionless constants, and the modified model could yield a significant improvement in modeling the pressure–sinkage relationship's response to plate dimensions (Reece, 1965). Recently, Spenko and Meirion-Griffith modified Bekker's equation by accounting for both the wheel width and diameter for wheeled mobile robots (Meirion-Griffith and Spenko, 2013, 2011). No matter whether the pressure-sinkage model is exploited for terrestrial vehicles or wheeled mobile robots, the normal stress is always expressed as a power function of the wheel sinkage.

Due to the complexity of normal stress's expression, the wheel-soil interaction force and torque equations are not amenable to closed-form integration. To estimate terrain parameters in real time, an algorithm that relied on a linear approximation of the normal stress equation was proposed (Iagnemma et al., 2004). Utilizing the estimated terrain parameters, a rough terrain control methodology was presented to improve ground traction and reduce power consumption for wheeled mobile robots (Iagnemma and Dubowsky, 2004).

The goal of this paper is to develop a linear normal stress as the function of the wheel sinkage. The rest of this paper mainly contains four parts. Part 2 concentrates on the introduction of wheel-soil interaction mechanics in skid. The new reference of wheel sinkage is presented in Part 3. The linear normal stress model is established in Part 4. Finally, Part 5 described the results of experimental validation using two wheels and on two sands.

2. Wheel-soil interaction modeling in skid

Skid between wheel and soil generates wheel resistance forces. The definition of skid ratio is shown in Eq. (1), where r denotes wheel radius, v and ω denote wheel forward velocity and angular velocity, respectively.

$$s_d = (v - r\omega)/v = 1 - r\omega/v \ (0 \leqslant s_d \leqslant 1) \quad (1)$$

Force diagram of a wheel in skid is shown in Fig. 1, where W is the wheel vertical load, F_V is the vertical force, F_R is the resistance force from the soil, θ_1 is the entrance angle, θ_2 is the leaving angle, z_1 is the wheel vertical sink-