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Research paper

Overall properties of particulate composites with periodic microstructure in second strain gradient theory of elasticity

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ABSTRACT

In the context of Mindlin's second strain gradient theory, a homogenization scheme for the determination of the effective overall properties of particulate composites with periodic microstructure is developed in this paper. This homogenization method is based on the direct application of an equivalent inclusion method adapted to second strain gradient theory in such a manner that the consistency conditions, in addition to the stress field, are applied to the double stress and triple stress fields of the representative unit cell of composite and its corresponding inclusion problem. Subsequently, by equating the potential energy of the composite material with that of an equivalent homogeneous material, the effective overall elastic moduli of the composite are determined. Moreover, it is shown that the energy expression of the equivalent homogeneous material includes two additional parameters, namely, the cohesion-mismatch-induced initial stress and the cohesion-mismatch-induced surface tension, both arising from the mismatch in the cohesion moduli of the matrix and the reinforcing particles of the composite. The effects of particle size and particle–matrix interface manifest themselves in the formulations and the obtained results of the paper.

1. Introduction

The concepts of eigenstrain and inclusion are effective tools in micromechanics of solids for the understanding of a wide spectrum of phenomena such as thermal expansion, phase transformation, or plastic strains. These concepts, on the other hand, play key roles in the micromechanical schemes exploited for the determination of the effective overall properties of heterogeneous materials. Among the pioneer studies concerned with such concepts should be mentioned the celebrated paper of Eshelby (1957) who addressed the problem of an isolated ellipsoidal inclusion embedded in an infinite medium. Eshelby showed that, if a uniform distribution of eigenstrain is prescribed in an ellipsoidal inclusion, then the induced disturbance strain field inside the inclusion will be uniform (Eshelby, 1957). He accordingly introduced a fourth-ranked tensor known as Eshelby's tensor that relates the components of the disturbance strain tensor to those of the eigenstrain tensor. As a result, a new prospect was opened to researchers and, afterwards, abundant efforts were made to investigate similar and relevant problems. Solutions to a variety of problems including periodically distributed inclusions as well as isolated inclusions are available in the book by Mura (1987).

The other significant contribution of Eshelby is to establish a method for the determination of the elastic field of an isolated inhomogeneity embedded in an infinite medium subjected to a remote applied load (Eshelby, 1961). According to such method, an inhomogeneity is replaced by an inclusion with the same shape and size and, then, a proper distribution of homogenizing eigenstrain is prescribed within the inclusion in such a manner that the stress fields of the two problems become identical to each other. Such an interesting idea, referred to as the equivalent inclusion method, has broadly been used to study the mechanical behavior of heterogeneous materials, particularly fibrous and particulate composites. In fact, it should be stated that the equivalent inclusion method is a cornerstone of the micromechanical approaches developed for the estimation of the effective overall properties of composites. From a general point of view, such approaches can be categorized into two groups: (*i*) the average-field methods and (*ii*) the homogenization methods. The average-field methods, including Mori–Tanaka theory (Mori and Tanaka, 1973) and self-consistent theory (Hill, 1963; Budiansky, 1965), are based on the fact that the effective overall properties of a heterogeneous body are connected with the volume average of its elastic fields where the solution to these fields is approximated by that of an isolated inhomogeneity embedded in an unbounded medium. While the average-field methods are often admissible to study dilute composites, the homogenization methods make it possible to obtain a rigorous pre-

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diction of the effective overall properties of composites consisting of densely packed inhomogeneities. For example, let us mention the papers by Nemat-Nasser and coworkers (Nemat-Nasser and Taya, 1981; Nemat-Nasser et al., 1982; Iwakuma and Nemat-Nasser, 1983) who employed a homogenization scheme for the determination of the macroscopic behavior of particulate composites with periodic microstructure. According to their approach, the equivalent inclusion method is applied to the representative unit cell of composite and, consequently, the elastic field of the unit cell is determined. Then, by setting the strain energy of the composite medium equal to that of an equivalent homogenous body, the effective overall elastic moduli of the composite are calculated. Along this line of thought, one may find several studies in the literature concerned with different aspects of the macroscopic mechanical behavior of composite materials (Rodin, 1993; Wu et al., 1997; Shodja and Roumi, 2005, 2006).

It should be remarked that all of the aforementioned investigations are in the framework of classical theory of elasticity. Even though the effectiveness and accuracy of this theory for a wide range of engineering applications have been proved, nevertheless there exist some critical limitations to its application, in particular, to small-scale problems where size effect has an important role in the mechanical behavior of materials (Kouzeli and Mortensen, 2002; Cho et al., 2006; Vollenberg and Heikens, 1989). To overcome such a drawback, a variety of higher-order theories including Cosserat theory (Cosserat and Cosserat, 1909), couple stress theory (Mindlin and Tiersten, 1962), nonlocal theory (Eringen and Edelen, 1972), and strain gradient theories (Toupin, 1962; Mindlin, 1965) have thus far been introduced and developed. It is worthwhile to mention here that, even though all of the higher-order theories are somehow capable of accounting for a size effect, many of them, however, may lead to quite different solutions for a single problem. Hence, it should be stated that the validity of predictions of these theories is in doubt unless supported by experimental findings.

By employing the higher-order theories, several authors have dealt with various problems related to inclusions and inhomogeneities. For example, Cheng and He (1995) determined Eshelby's tensors for a spherical inclusion embedded in a Cosserat medium. Moreover, they applied an equivalent inclusion method to the problem of inhomogeneity in the framework of micropolar elasticity and derived the associated equivalency conditions. Ma and Hu (2006) obtained analytical forms for Eshelby's tensors of an ellipsoidal inclusion in a micropolar medium. Liu and Hu (2004) derived expressions for Eshelby's tensors of a spherical inclusion in a microstretch medium. Zheng and Zhao (2004) obtained Eshelby's tensors for a spherical inclusion embedded in a couple stress body. Gao and Ma, by employing a simplified version of first strain gradient theory, provided analytical expressions for Eshelby's tensors of ellipsoidal (Gao and Ma, 2010) and cylindrical inclusions (Ma and Gao, 2010). The solutions obtained in the context of the higher-order theories in these studies have demonstrated the effect of inclusion size on its elastic state and stated that such an effect for inclusions with smaller dimensions is more significant.

While all of the latter-mentioned studies are concerned with the problem of an isolated inclusion/inh-omogeneity, the higher-order theories have, in addition, been utilized for the examination of the overall behavior of composite materials by several authors. For example, Yuan and Tomita (2001) employed a homogenization method to predict the macroscopic behavior of a heterogeneous Cosserat material with periodic microstructure. Xun et al. (2004), by using the average-field theory, determined the effective in-plane shear and bulk moduli of a micropolar composite medium having coated fibers. Haftbaradaran and Shodja (2009), by employing Mori–Tanaka theory, estimated the overall anti-plane shear moduli of a couple stress matrix with unidirectional elliptic cylindrical fibers. Zhang and Sharma (2005), by postulating a simplified form of second strain gradient theory, extended the equivalent inclusion method to determine the elastic state of a spherical inhomogeneity and, subsequently, within Mori–Tanaka scheme obtained expressions for the effective overall properties of a composite medium reinforced with such inhomogeneities. Ma and Gao (2014), in the framework of a simplified version of first strain gradient theory and by using Mori–Tanaka theory, developed a method for the determination of the effective overall elastic moduli of composite matrices reinforced by spherical, cylindrical or ellipsoidal particles.

The purpose of the present study is to extract estimates of the effective overall properties of a particulate composite with periodic microstructure in the framework of Mindlin's second strain gradient theory. To this end, first an equivalent inclusion method adapted to such theory is applied to the representative unit cell of the composite and, then, the corresponding consistency conditions are derived. By utilizing Fourier series expansion method, a solution for the elastic state of the problem is obtained. Subsequently, by introducing a homogenization technique consistent with second strain gradient theory, the effective overall elastic moduli of the composite are determined. In light of the features of second strain gradient theory, the proposed approach in this paper is expected to be capable of capturing simultaneously the effects of particle size and particle–matrix interface. Hence, the results and discussion section of this paper is dedicated to the calculation of the overall properties of particulate composites reinforced with nano-sized particles. The discrepancy between the results of the proposed approach and those of classical elasticity for such composites is quite evident.

2. A brief review on the fundamentals of second strain gradient theory

In Mindlin's second strain gradient theory of elasticity (Mindlin, 1965), the potential energy density function W of an elastic solid medium is assumed to be a function of the first and second gradients of the strain tensor in addition to the strain tensor itself. In this theory, W is expressed as

$$W \equiv W(e_{ij}, e_{ijk}, e_{ijkl}) \tag{1}$$

in which e_{ij}, e_{ijk}, and e_{ijkl} are, respectively, the components of the strain, double strain, and triple strain tensors that are related to the displacement component u_i via

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{2a}$$

$$e_{ijk} = u_{k,ij}, \tag{2b}$$

$$e_{ijkl} = u_{l,ijk}. \tag{2c}$$

For a hyperelastic material, the energy density function W can be expanded in the form (Ojaghnezhad and Shodja, 2013)

$$W = \frac{1}{2} C_{ijkl}\, e_{ij}\, e_{kl} + F_{ijklmn}\, e_{ij}\, e_{klmn} + \frac{1}{2} G_{ijklmn}\, e_{ijk}\, e_{lmn} + \frac{1}{2} H_{ijklmnpq}\, e_{ijkl}\, e_{mnpq} + B_{ijkl}\, e_{ijkl} \tag{3}$$

where the components of the elastic moduli tensors C_{ijkl}, F_{ijklmn}, G_{ijklmn} $H_{ijklmnpq}$, and B_{ijkl} for an isotropic material are given by

$$C_{ijkl} = \lambda\, \delta_{ij}\, \delta_{kl} + \mu\left(\delta_{ik}\, \delta_{jl} + \delta_{il}\, \delta_{jk}\right), \tag{4a}$$

$$F_{ijklmn} = \frac{1}{3} c_1\, \delta_{ij}\, \delta_{klmn} + \frac{1}{6} c_2\left(\delta_{jnmlki} + \delta_{jnklmi}\right) + \frac{1}{6} c_3\left(\delta_{iknmlj} + \delta_{jmnkli}\right), \tag{4b}$$