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Analytical study on stress-induced phase transitions in geometrically graded shape memory alloy layers. Part II: Analyses on geometrical shapes, loading procedures and boundary conditions

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ABSTRACT

Based on the asymptotic ODE system derived in part I of this series of papers, the mechanical behaviors of geometrically graded shape memory alloy (SMA) layers with different shapes, loading procedures and boundary conditions are studied in the current paper. First, for SMA layers with concave and convex shapes, the stress-strain curves during a full loading cycle are plotted based on the analytical solutions to the asymptotic ODE system, which can capture most of the experimental features. The effects of width ratio on the responses of the SMA layers are also discussed. Second, corresponding to the inner loops of the stress-strain curves, the configurations and phase states of the SMA layers with the different shapes are simulated. The underlying mechanisms responsible for the experimental features are also investigated. Third, the effects of boundary conditions are studied, where the free end boundary conditions, the mixed boundary conditions and the clamped boundary conditions are taken into account. The corresponding analytical solutions are derived and the responses of the SMA layers are simulated, which are compared with each other to reveal the effects of boundary conditions. The results obtained in the current paper further demonstrate the validity of the analytical approach for studying the phase transitions in geometrically graded SMA specimens.

1. Introduction

In this series of two papers, we aim to propose an analytical approach to study the stress-induced phase transitions in geometrically graded shape memory alloy (SMA) layers. In part I of this series of papers (Liu et al., 2016), we first formulated the governing PDE system for slender SMA layers within a two-dimensional (2D) setting, which contains the mechanical equilibrium equation and the phase transition criteria. Then, by using the coupled series-asymptotic expansion method (Dai and Cai, 2006; Cai and Dai, 2006), we derived one single ODE from the governing system, which represents the asymptotic relationship between the axial loads and the leading-order term of the axial strain (cf. Eq. (24) in part I). By retaining the original variables, this ODE can be written as

$$-\frac{T}{2EA}+V-s_1\alpha_0-(s_1+s_2)V\alpha_0+s_1(s_1+s_2)\alpha_0^2-\frac{1}{3}s_1A^2\alpha_1 +\frac{T\xi_2A_{XX}}{4(\xi_1^2-\xi_2^2)}-\frac{1}{3}A^2V_{XX}+\frac{1}{6}A^2(2s_1+s_2)\alpha_{0XX}=0, \quad (1)$$

eq1)where $V(X)$ is the leading-order term of the axial strain, T is the resultant force acting on the cross section of the layer, $\alpha_0(X)$ and $\alpha_1(X)$ are the first two terms of the phase state variable in the series expansion, $A(X)$ represents the half width-length ratio of the layer in the reference configuration, s_1 and s_2 are two characteristic stretches associated with the phase transformation, ξ_1 and ξ_2 are the first-order elastic moduli (cf. Appendix A in part I) and $E=(\xi_1^2-\xi_2^2)/\xi_1$ plays the role of Young's modulus in 2D case.

The relationships between the axial strain $V(X)$ and the phase state functions $\alpha_0(X)$ and $\alpha_1(X)$ should be derived from the phase transition criteria (cf. Eq. (7) in part I). However, it is well known that for SMAs the correspondence between the axial strain and the phase state is not unique, which directly depends on the loading

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history (cf. Rajagopal and Srinivasa, 1999). In part I, we considered the pure loading and pure unloading processes and derived the following relationships between $\alpha_0(X)$, $\alpha_1(X)$ and $V(X)$

$$\alpha_0^{\pm}(X) = \begin{cases} 0, & \text{if } 0 \le V < V_0^{\pm}, \\ L_1^{\pm} + L_2^{\pm} T A_{XX} + L_3^{\pm} V + L_4^{\pm} V^2, & \text{if } V_0^{\pm} \le V \le V_1^{\pm}, \\ 1, & \text{if } V > V_1^{\pm}, \end{cases} \tag{2}$$

$$\alpha_1^{\pm}(X) = \begin{cases} 0, & \text{if } 0 \le V < V_0^{\pm} \text{ or } V > V_1^{\pm}, \\ L_5^{\pm} V_{XX}, & \text{if } V_0^{\pm} \le V \le V_1^{\pm}. \end{cases} \tag{3}$$

where the superscripts $' \pm '$ represent the loading and the unloading processes respectively, and $V_0^{\pm}$ and $V_1^{\pm}$ are some critical strains corresponding to the intersection points of the different regions. The formulas for $V_0^{\pm}$, $V_1^{\pm}$ and the coefficients $L_i^{\pm}$ $(i = 1, \cdots, 5)$ can be found in Appendix C of part I. By substituting (2) and (3) into (1), we obtained the following asymptotic ODE system corresponding to the pure loading and pure unloading processes

$$\begin{cases} -\dfrac{T}{2EA} + \dfrac{T\xi_2 A_{XX}}{4(\xi_1^2 - \xi_2^2)} + V - \dfrac{1}{3}A^2 V_{XX} = 0, & \text{if } 0 \le V < V_0^{\pm}, \\ M_0^{\pm} + \dfrac{M_1^{\pm} T}{-2EA} + M_2^{\pm} T A_{XX} - M_3^{\pm} V & \\ \quad + M_4^{\pm} V^2 - \dfrac{1}{3} A^2 V_{XX} = 0, & \text{if } V_0^{\pm} \le V \le V_1^{\pm}, \\ -\dfrac{T}{2EA} + \dfrac{T\xi_2 A_{XX}}{4(\xi_1^2 - \xi_2^2)} + (1 - s_1 - s_2)(V - s_1) & \\ -\dfrac{1}{3}A^2 V_{XX} = 0, & \text{if } V > V_1^{\pm}, \end{cases} \tag{4}$$

where the coefficients $M_i^{\pm}$ depend on the elastic moduli and some other material parameters (cf. Appendix C of part I). The three ODEs in (4) correspond to the austenite, the martensite and the phase transition regions in a SMA layer, respectively. At the intersection points $V = V_0^{\pm}$ and $V_1^{\pm}$, we have proposed the following connection conditions

$$V_+ = V_-, \quad (V_X)_+ = (V_X)_-. \tag{5}$$

In part I, to show the validity of the asymptotic ODE system (4), we have studied the mechanical behaviors of linearly tapered SMA layers subject to free end boundary conditions. In the current paper, based on the asymptotic Eqs. (1) and (4), we shall further study the mechanical behaviors of geometrically graded SMA layers with different shapes, loading procedures and boundary conditions. The asymptotic equations will be solved by using suitable analytical (asymptotic) method. The obtained solutions will be used to simulate the mechanical behaviors of the SMA layers and to reveal the underlying mechanisms responsible for the key experimental features.

In Shariat et al. (2013a; 2013b), the SMA strips with linearly tapered, parabolic concave and convex shapes were adopted in the tensile loading tests. It had been found that the linearly tapered specimen exhibits rather linear evolution of stress, while the other two kinds of specimens show curved stress gradients over the phase transition processes. As the mechanical behaviors of linearly tapered SMA layers have been studied in part I, we shall further study the SMA layers with parabolic concave and convex shapes in Section 2 of the current paper. Some specific forms of $A(X)$ will be chosen to represent the geometrical shapes of the SMA layers, which contain the parameter to represent the minimum-maximum width ratio. The asymptotic ODE system (4) will be solved by using the WKB method. Based on the obtained solutions, the responses of the SMA layers with the different shapes can be predicted. It will be shown that the key features of the experimental results can be captured by the obtained analytical results. The influence of the width ratio on the response of the SMA layers will also be analyzed.

The responses of SMA specimens during the pure loading and pure unloading processes constitute the outer loops of the stress-strain curves. In this paper, we shall further consider the loading processes corresponding to the inner loops of the stress-strain curves, where the reverse loading processes start just after the phase transitions in the specimens are partially completed. The mechanical behaviors of SMA specimens associated with the inner loops (or in more complicated cyclic loading procedures) have been investigated in many experiments (e.g., Lin et al., 1994; Lexcellent and Tobushi, 1995; Feng and Sun, 2006; Shariat et al., 2013a; Rao et al., 2014) and some important features have been observed. For example, the SMA specimen exhibits elastic response when the stress value lies between the upper and the lower parts of the outer loops, and the reverse (forward) phase transformation starts when the stress value attains the lower (or upper) parts of the outer loops. In the authors' previous work (Wang and Dai, 2012), the inner loops of the stress-strain curves of homogeneous SMA specimens have been studied and some of the underlying mechanisms have been revealed. In Section 3 of this paper, we shall further study the inner loops of the stress-strain curves obtained from the geometrically graded SMA layers. With the different geometrical shapes, the mechanical responses of the SMA layers associated with the inner loops will be predicted and compared with the experimental results. The underlying mechanisms responsible for the experimental features will be further identified.

The experimental results have revealed that the boundary conditions have obvious influences on the responses of SMA specimens (cf. Shaw and Kyriakides, 1995; Tse and Sun, 2000). In the authors' previous works (Dai. and Wang, 2009; Dai et al., 2009), the effects of boundary conditions on stress-induced phase transitions in slender hyperelastic specimens have been studied. In Section 4 of the current paper, we shall further investigate the effects of end boundary conditions on the responses of geometrically graded SMA layers. Besides the free end boundary conditions considered in part I, we shall also consider the clamped boundary conditions and the mixed boundary conditions (free end-clamped boundary conditions). Due to the restrictions of the boundary conditions, the configurations of the SMA layer and the phase state distributions in the layer will exhibit large differences. Correspondingly, different solving schemes need to be adopted to derive the analytical solutions. Based on the obtained analytical results, the responses of the SMA layer with the different boundary conditions are predicted and compared with each other to reveal the effects of boundary conditions.

Finally, some conclusions will be drawn in Section 5.

2. Mechanical behaviors of SMA layers with concave and convex shapes

To investigate the effects of geometrical shape on the response of SMA specimens, in this section we shall study the stress-induced phase transitions in SMA layers with different shapes. In Shariat et al. (2013a; 2013b), SMA strips with parabolic concave and convex shapes have been adopted in the tensile loading tests. The mechanical responses of these SMA strips were measured and compared with the responses of linearly tapered SMA strips. It was observed that during the phase transition processes, the linearly tapered specimen exhibits rather linear evolution of stress, while the two parabolic specimens show curved stress gradient. It can also be found that the concavity and convexity of the strips affect the curvature direction of stress gradient for the stress-induced phase transitions. Besides that, tensile loading tests have also been conducted on parabolic SMA strips with different minimum-maximum width ratios (the maximum widths of