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Research paper

Detecting precursors of localization by strain-field analysis

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ABSTRACT

Localization in the deformation field, even though initiated locally at the microscopic scale, leads upon increased deformation to fracture at the macroscopic scale, thereby violating the separation of length scales. Localization and damage can be accounted for in macroscopic modeling by appropriate enrichments at that level, however doing so requires (i) detecting the onset of localization prior to its actual occurrence and (ii) quantifying the kinematical characteristics of the localization band. This paper serves that goal. A methodology is developed to analyze the evolution of strain- and displacement-fields during deformation. A key ingredient in this analysis is the use of the Minkowski functionals (also known as intrinsic volumes, quermass integrals, or curvature integrals) from integral geometry, to detect emerging patterns in thresholded strain- and displacement-fields. Doing so, the onset of localization in the microstructure is detected as the emergence of a correlated and narrow pattern of high strains, prior to the actual loss of material stability. Furthermore, the developed localization band is characterized in terms of a weak displacement discontinuity, incorporating the width and direction of the band. The developed methodology uses kinematical fields only, and is therefore applicable to both numerical and experimental deformation-field data. For illustration purposes, numerical data from a finite-element simulation of a deformed voided microstructure is used, without any loss of generality.

1. Introduction

Multiscale methods provide an essential contribution to bridging scales in mechanics of materials and material science. Nowadays, they are widely applied in designing, engineering and processing of advanced materials, e.g. composite materials, advanced alloys, biological materials. In recent years, various multiscale methods have been proposed to tackle scale bridging, e.g. asymptotic and homogenization methods (Cailletaud et al., 2003), heterogeneous multiscale methods (E et al., 2007; Chen, 2009), variational multiscale methods (Hughes et al., 1998), computational homogenization (Kouznetsova et al., 2001; Smit et al., 1998; Miehe et al., 1999; Geers et al., 2010; Schröder and Hackl, 2014; Oller, 2014) and experimental studies (Efstathiou et al., 2010). However, several classes of engineering problems represent cases in which scales strongly interact. In other words, there is no clear separation of length scales, and hence a clear distinction of phenomena into fine- or coarse-scale features is cumbersome, if not even impossible. A prominent example in this respect is material damage. Initiation and growth of damage is typically associated with the emergence of narrow regions with localizing strains at the microscale (Coenen et al., 2012b; Nguyen et al., 2011). While the progressive degradation of a material starts at the micro-scale, it gradually propagates to the macro-scale until material stability is lost, resulting in overall failure, i.e. fracture. As damage spans a wide range of length scales, it intrinsically violates scale separation and compromises the applicability of existing multiscale methods. In turn, this implies that, depending on the progression of the damage, the material model that is used to study the macroscopic behavior needs to be enriched. To that end, it is mandatory to devise a methodology that detects the necessity for an enrichment of the macroscopic model prior to the actual loss of stability. Therefore, an appropriate characterization of pattern formation in the microscale features is required in order to identify a precursor of localization.

Several studies have investigated strain localization by applying prototypical modeling, numerical simulations and experimental investigations. Various experimental techniques allow to investigate deformation and strain localization of materials at various length scales, including non-contact optical and interferometric methods

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(Cloud, 1998), atomic force microscopy (Tanaka et al., 2007; Man et al., 2002), transmission electron microscopy (TEM) (Saito et al., 2005), scanning electron microscope (SEM) (Crostack et al., 2001; Sutton et al., 2007b; Tanaka et al., 2011) and digital image correlation (DIC) (Pan et al., 2009; Marty et al., 2015; Arikawa et al., 2011; Kammers and Daly, 2013; Sutton et al., 2007a). Several advanced numerical methods are dedicated to micro-mechanical modeling that incorporates damage (Uthaisangsuk et al., 2009; Tekoglu and Pardoen, 2010; Legarth and Niordson, 2010; Ghosh et al., 2009; Kim and Lee, 2010) for which a continuous-discontinuous homogenization scheme is required to capture localization (Coenen et al., 2011, 2012b; Nguyen et al., 2012, 2011; Bosco et al., 2014; T. Belytschko, 2010; Ji et al., 2015; Paul and Kumar, 2012). Note that identifying material instabilities may be involved, see e.g. Benallal and Comi (1996); Szabó (2000); Benallal et al. (2010); Altmeyer et al. (2013). Here, the onset of localization will be captured naturally on the basis of the observed kinematics.

Localization occurs as a rapid collective growth and concentration of the microfluctuations in the strain field, culminating into a narrow high-gradient region in the sample, typically accompanying strain softening and thereby inducing material instability. Localization generally manifests itself as an irregular band of intense strains crossing the microstructure and provoking the emergence of a pronounced displacement variation. The latter is typically described by a weak discontinuity (Bosco et al., 2014; Liu, 2015). A weak discontinuity divides the micro-scale into regions of small and large displacements, respectively, separated by a smooth but pronounced transition. At the macro-scale, this may also be captured as a weak discontinuity or a discrete jump in the displacement field. Both cases require that the kinematical characteristics of the strong or weak discontinuity are properly quantified and embedded into the (thereby extended) macroscopic description to account for the localization, see e.g. Vernerey et al. (2007, 2008) and Wang and Lee (2010).

There exists a strong demand for a technique, applicable both to numerical and experimental strain- and displacement-fields, that efficiently analyzes micro-fluctuations in order to detect localization patterns, prior to the loss of material stability, thereby signaling when an enrichment of the macroscopic model (Coenen et al., 2012b) becomes necessary. To predefine the onset of localization in a deforming microstructure, digital image analysis is applied to investigate developing patterns in the micro-fluctuations in the deformation field. The latter makes use of a set of sequential snapshots of strains or displacements in the deforming microsample. These snapshots may be obtained from an experimental analysis, e.g. using DIC, or by numerical modeling at the microscale. The resulting micro-fluctuation field is decomposed into its stochastic (i.e. uncorrelated) part and spatially correlated part, respectively. Strain localization typically entails evolving correlated patterns. The kinematical characteristics of these patterns need to be qualified and quantified, necessitating a systematic analysis of the overall morphology and topology of the patterns.

The goals of this study are (i) to detect the precursors of localization and (ii) to quantify the kinematical parameters required to enrich the macroscopic description in the presence of localization, based on the analysis of micro-fluctuations in the strain- and displacement-fields. Analysis of complex spatially fluctuating structures is a relevant subject in statistical physics, for which several methods and tools have been proposed. The present article employs the so-called Minkowski functionals (Hadwiger, 1957; Santalò, 1976; Schneider, 1993; Weil, 1983; Munkres, 2000; Ohser and Mücklich, 2000), also known as intrinsic volumes (quermass integrals, curvature integrals), to analyze the evolving patterns in the micro-fluctuation field. Minkowski functionals represent a set of morphological descriptors describing the geometry of objects using global integral quantities, in contrast to differential-geometric tools that provide local information. Analysis by Minkowski functionals finds wide application in physics, soft matter science, and medicine (Mecke and Stoyan, 2000; Petri et al., 2013; Hütter, 2003; Arns et al., 2010; Li et al., 2012).

The paper is organized as follows. Section 2 reviews the Minkowski functionals as a mathematical tool for analyzing the morphology of digital images. A complete procedure for analyzing the micro-fluctuation field in order to identify correlated patterns leading to localization is presented in Section 3. Section 4 is dedicated to the kinematical enrichment in the macro-scale model in order to account for the key characteristics of the localization band. Finally, conclusions are presented in Section 5.

2. Analyzing digital images: Minkowski functionals

The evolution of any scalar field, e.g. the equivalent total strain field, can be represented by a set of digital snapshots at discrete time instants. Digital images of the scalar field can be obtained by either numerical simulations or experimental techniques, thereby making the proposed approach applicable to both types of analysis. The evolution of the patterns in the scalar field of interest is captured and described by Minkowski functionals.

Minkowski functionals are mathematically represented as integral measurements of shape (Hadwiger, 1957; Santalò, 1976; Schneider, 1993; Weil, 1983; Munkres, 2000; Ohser and Mücklich, 2000). In d-dimensional space $\mathbb{R}^d$, there are $d+1$ scalar Minkowski functionals W_ν ($\nu = 0, .., d$) that describe a domain Ω with regular boundary $\partial\Omega$. The present study is restricted to $\mathbb{R}^2$ space, i.e. to the analysis of 2D images. The three scalar Minkowski functionals are then represented as follows,

$$W_0(\Omega) = \int_\Omega \mathrm{d}\Omega, \quad W_1(\Omega) = \frac{1}{2}\int_{\partial\Omega} \mathrm{d}r, \quad W_2(\Omega) = \frac{1}{2}\int_{\partial\Omega} k\mathrm{d}r, \tag{1}$$

with $\mathrm{d}\Omega$ an infinitesimal area element, $\mathrm{d}r$ a one-dimensional line element on the boundary $\partial\Omega$, and k the (local) principle curvature of the boundary. These three Minkowski functionals W_ν are related to the area (W_0), boundary length (W_1), and Euler characteristic (W_2) that describes the topology in terms of the connectivity of the structures in the domain $\Omega \subset \mathbb{R}^2$.

While the above definitions (1), particularly the ones for W_1 and W_2, refer to a differentiable smooth boundary, alternative formulations have been developed (Mantz et al., 2008; Michielsen and De Raedt, 2001) that are applicable to the case of pixelized (2D) and voxelized data (3D). As a consequence, Minkowski functionals are widely applied to analyze texture of gray-scale 8-bit digital images. Gray-scale images are presented by a set of pixels with intensity i_p in the range $i_\mathrm{p} \in [0, 255]$. Intensities $i_\mathrm{p} = 0$ and $i_\mathrm{p} = 255$ correspond to the black and white color, respectively. A threshold $i_\mathrm{th} \in [0, 255]$ divides the image into foreground and background patterns: If the intensity of the pixel $i_\mathrm{p} > i_\mathrm{th}$ then it is re-assigned to the maximum $i_\mathrm{p} = 255$, otherwise it is re-assigned to $i_\mathrm{p} = 0$. Doing so, a black-and-white binary image is obtained, which amounts to defining the pixel-equivalent of Ω in (1). The black and white patterns in the binary image are referred to as background and foreground patterns, respectively. The Minkowski functionals W_ν for gray-scale digital images are calculated on the thresholded variants, and are therefore functions of the threshold value i_th, $W_\nu = W_\nu(i_\mathrm{th})$, quantifying the area, boundary length and connectivity (Euler characteristic) of the black structures for each value of i_th. Increasing the threshold i_th results in an increase of the background area W_0. Corresponding changes in the boundary length are reflected by W_1. The Euler characteristic, represented by the third functional W_2, is defined as the difference between the number of disconnected background components (black structures)