Mechanics of Materials 104 (2017) 60–72

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Cascade continuum micromechanics model for the effective permeability of solids with distributed microcracks: Self-similar mean-field homogenization and image analysis

Jithender J. Timothy, Günther Meschke*

Institute for Structural Mechanics, Ruhr University Bochum, 44801, Germany

ARTICLE INFO

Article history:
Received 24 March 2016
Revised 11 October 2016
Available online 19 October 2016

Keywords:
Effective permeability
Microcracks
Porous materials
Homogenization
Recurrence
Continuum micromechanics

ABSTRACT

The transport and fluid flow in heterogeneous materials such as rocks, ceramics and concrete with a distributed random microcrack network is strongly influenced by the density and the topology (distribution and connectivity) of microcracks. The overall fluid flow characteristics of such microcracked solids can be quantified in terms of an effective permeability. In the paper, a semi-analytical formulation for the effective permeability is proposed within the framework of the mean-field homogenization method using the cascade continuum micromechanics model considering long range and short range interactions. We compare model predictions of the percolation threshold i.e. critical volume fraction of microcracks beyond which a solid with distributed microcracks becomes permeable, using results from numerical simulations. The model reveals a new perspective into the self-similar characteristics of the microcrack morphology near the threshold volume fraction of microcracks at which the microcrack structure changes from multiple disconnected microcracks to a connected self-similar microcracked structure.

1. Introduction

Heterogeneous geological materials such as rocks and engineered porous materials such as ceramics, cementitious materials, masonry and concrete are characterized by a heterogeneous microstructure, which generally includes, besides aggregates and the pore space, also distributed microcracks. These microcracks have different origins and are resulting from geological metamorpohis processes, shrinkage or mechanical, hygral and thermal loading processes (see Fig. 1). Distributed cracks, further denoted as microcracks in the paper, strongly modify the fluid flow characteristics through the porous material by enhancing the connectivity of the pore-structure and providing additional fluid flow pathways. Once the permeability of the porous matrix is known, the influence of the microcracked microstructure (see Fig. 2) on the fluid flow can be characterized by an effective permeability $\mathbf{k}_{eff}$.

Fluid flow in quasi-brittle porous materials is generally described by the Darcy law (Darcy, 1856). According to this law, the influence of the porous microstructure on the fluid flow is characterized by the permeability $k[m^2]$, which, in an average sense at a macroscopic scale, represents the influence of the constituents at lower spatial scales on the transport properties of the material. This paper focusses on the role of networks of distributed random microcracks in materials that are either impermeable or porous and its influence on the effective permeability. The permeability of the intact porous material itself is assumed to be known *a priori*.

Modeling the effective properties of heterogeneous materials is a classical problem of physics and engineering. Idealizing the heterogeneous microstructure in terms of multiple phases, the simplest approach is the Voigt (Voigt, 1889) average of the permeability of the intact porous material and the intrinsic permeability of the microcrack, that implies a parallel arrangement of the individual phases. This model is strictly applicable only to systems, where the microcracks are all connected to each other, parallel and infinitely long. However, for an isotropic distribution of finite sized microcracks generally encountered in cementitious, ceramics and geological materials, the microcrack network and distribution introduce an additional 'structural' effect called the percolation threshold (Broadbent and Hammersley, 1957). The percolation threshold is the critical microcrack density (or the volume fraction of the microcracks), below which the number of microcracks to form a connected microcrack network is insufficient. This effect on fluid flow, from logical reasoning, is clearly proportional to the ratio (material contrast) of the intrinsic properties of the microcrack and the intact material. The Voigt model does not consider this effect.

* Corresponding author.
E-mail addresses: timothy.jithenderjaswant@rub.de (J.J. Timothy), guenther.meschke@rub.de (G. Meschke).

http://dx.doi.org/10.1016/j.mechmat.2016.10.005
0167-6636/

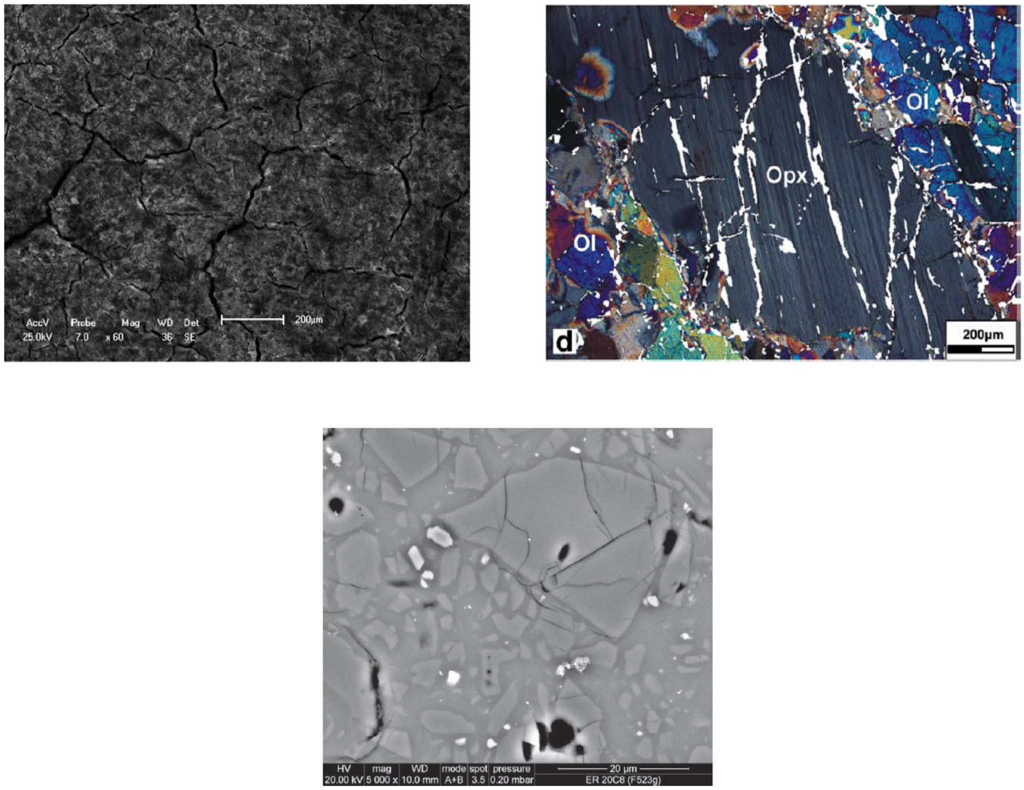

Fig. 1. SEM images of microcracks on the surface of a concrete sample at a temperature loading of 200°C (top left), microcracks in ultrabasic rocks subject to compressive stresses (top right), microcracks in ceramics as a result of fast cooling (bottom) (reprinted from (Wang et al., 2005; Rigopoulos et al., 2011; Gilabert et al., 2014) with permission from Elsevier).

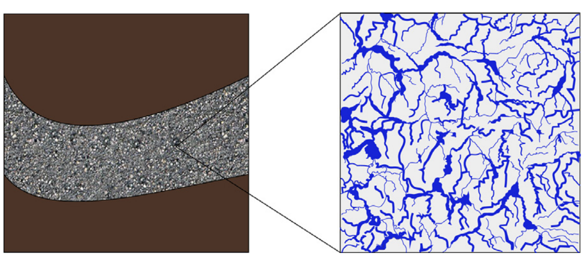

Fig. 2. Illustration of a heterogeneous structure of rocks (left) and a corresponding microcracked REV Ω_{REV} with a fully saturated microcrack network.

Hence, the problem of estimating the effective properties requires an appropriate averaging of the intrinsic properties of the matrix material and the microcracks, simultaneously taking into consideration the percolation characteristics associated with the specific topology of the microcrack network. To this end, a wide variety of methods, such as the mean-field homogenization methods, percolation theory and renormalization methods have been proposed. These methods are summarized briefly below.

The mean-field homogenization method, also denoted as continuum micromechnics (Hashin, 1983; Nemat-Nasser and Hori, 1999; Zaoui, 2002), is a blanket term for a family of schemes that estimate the effective properties of multiphase materials by characterizing the mean field perturbation due to a particular heterogeneity using localization tensors (Hill, 1963; 1965) in conjunction with appropriate definitions of the far-field boundary-conditions to consider the interactions of multiple heterogeneities. The method finds applications in a wide range of multiphase media such as, cementitious materials (Lemarchand et al., 2003; Pichler et al., 2008), foams (Pichler and Lackner, 2013) and rocks (Saenger and Shapiro, 2002; Berryman and Hoversten, 2013; Zhu et al., 2016). Among the various schemes within this framework, the self-consistent scheme, which assumes that the matrix phase is that of the homogenized effective material is able to predict a percolation threshold. This idea goes back to the work of (Bruggeman, 1935; Landauer, 1952), finding applications in the elasticity (Hill, 1965; Sanahuja et al., 2007) and transport properties of media with pore-networks (Kirkpatrick, 1971; 1973; Koplik, 1981; David et al., 1990). Recently, the continuum version of this model was applied to fluid transport in isotropic and anisotropically microcracked materials (Fokker, 2001; Dormieux et al., 2006; Barthélémy, 2009; Pouya and Vu, 2012; Berryman and Hoversten, 2013). Under extreme conditions, when the contrast of the intrinsic properties of the material phases is infinite or zero (e.g. in case of microcracks in an impermeable intact material), the model predicts solutions with negative effective properties for a certain range of microcrack volume fractions. The recursive cascade micromechanics model (Timothy and Meschke, 2016b) originally proposed for estimating the effective diffusivity of porous materials satisfies the self-consistent equation and whose solutions are strictly positive. An extension of this model for the case of microcracked materials is used to estimate the effective permeability in this paper.

For materials with a periodic microstructure (Auriault et al., 2009), the double porosity approach is often used within the framework of formal asymptotic expansion to derive an effective homogenized permeability (Lewandowska et al., 2004).

While homogenization methods look at the problem of estimating the effective properties from the point of view of averaging, the percolation theory, as the name suggests, uses the percolation threshold as the starting point for estimating the effective properties using certain scaling laws (Efros and Shklovskii, 1976). These models study the connectedness of the phases as a function of the phase geometry and phase density or volume fraction using sta-