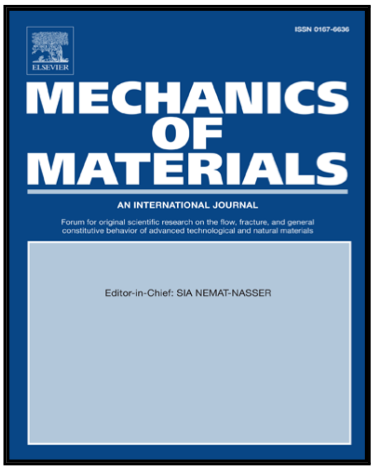
Accepted Manuscript



The Oblique Impact of a Rigid Sphere on a Power-Law Graded Elastic Half-Space

E. Willert , V.L. Popov

PII: S0167-6636(16)30571-3
DOI: 10.1016/j.mechmat.2017.03.019
Reference: MECMAT 2722

To appear in: *Mechanics of Materials*

Received date: 16 December 2016
Revised date: 15 March 2017
Accepted date: 28 March 2017

Please cite this article as: E. Willert , V.L. Popov , The Oblique Impact of a Rigid Sphere on a Power-Law Graded Elastic Half-Space, *Mechanics of Materials* (2017), doi: 10.1016/j.mechmat.2017.03.019

The Oblique Impact of a Rigid Sphere on a Power-Law Graded Elastic Half-Space

E. Willert[1], V.L. Popov[1,2,3]

[1]Berlin University of Technology, 10623 Berlin, Germany
[2]National Research Tomsk State University, 634050 Tomsk, Russia
[3]National Research Tomsk Polytechnic University, 634050 Tomsk, Russia

corresponding author: e.willert@tu-berlin.de

Abstract: The low-velocity oblique impact of a rigid sphere on a power-law graded elastic half-space is studied under the assumptions of elastic similarity and a constant coefficient of friction. The normal component of motion is determined analytically. The tangential problem is investigated numerically using the Method of Dimensionality Reduction. We find that the solution of the impact problem written in proper dimensionless variables is the same as in the homogeneous case. This solution therefore can possibly be generalised for arbitrary inhomogeneous material behaviour, if the Mindlin ratio has no spatial dependence. However, different physical ranges are possible for the dimensionless variables in the homogenenous and inhomogeneous cases, which is why, in the case of power-law grading, parameter combinations are possible, for which no kinetic energy is dissipated during the impact. The maximum contact pressures during normal impact can be significantly reduced by the usage of power-law grading.

Key words: Oblique Elastic Impacts, Friction, Power-Law-Graded Elastic Half-Space, Method of Dimensionality Reduction

1 Introduction

The increasing technological demand regarding variability, durability and other properties of materials led to the development of novel categories of high-performance composites, such as Functionally Graded Materials (FGM). Within a FGM the mechanical properties continuously vary in one or more directions. This is preferable over layered composites as it eliminates the possibility of delamination failure. The material properties of a FGM can be subtly adjusted and optimised for its application.

A typical dynamic loading protocol for a structure is impact loading. This is often used to test dynamic material responses or the durability against impact damage. Moreover, elastic particle collisions are the fundamentals of the dynamics and statistics of granular media [1].

Thereby, the history of rigorous contact-impact solutions goes back to the classical work of Hertz [2], who analysed the frictionless normal contact between two perfectly elastic bodies of parabolic shape and the associated impact problem. The tangential contact problem for elastically similar spheres was solved independently from each other by Cattaneo [3] and Mindlin [4]. They showed that the tangential tractions in the contact can be interpreted as a superposition of two Hertzian pressure distributions, but only studied a simple loading case of a constant normal force and an increasing tangential force. This was later generalised for various different loading protocols by Mindlin and Deresiewicz [5]. Based on the latter results Maw et al. [6] were the first to study the oblique impact of elastic spheres. They gave the governing parameters of the impact problem, but could not give a comprehensive solution due to computation restrictions.

In the past years Popov and his co-workers have shown, that the three-dimensional axisymmetric tangential contact problem for arbitrary loading protocols can be exactly