A thermo-mechanically coupled nonlinear viscoelastic–viscoplastic cyclic constitutive model for polymeric materials

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ABSTRACT

Thermo-mechanically coupled cyclic deformations often occur in polymeric components subjected to a cyclic loading. In this work, a framework of irreversible thermodynamics at small deformation is presented at first to model the thermo-mechanically coupled cyclic deformation of polymeric materials. Two inelastic mechanisms, i.e., viscoelasticity and viscoplasticity are considered, simultaneously. The thermodynamic state of the material is defined by certain state variables and the Gibbs free energy is decomposed into four parts, i.e., the instantaneous elastic, the viscoelastic, the viscoplastic and the ones related to temperature. The driving forces of the strain-like internal variables related to the viscoelasticity and viscoplasticity are deduced from the constructed Gibbs free energy and Clausius's dissipative inequality. The internal heat production and the spatio-temporal evolution equation of temperature are obtained by the first law of thermodynamics. Then, based on the experimental observations on the cyclic deformation of ultra-high molecular weight polyethylene (UHMWPE), a specific constitutive model is proposed by adopting some simplifications. Finally, the capability of the proposed model is verified by comparing the predicted results with the corresponding experimental ones of the UHMWPE. It is seen that the temperature- and rate-dependent cyclic deformation and the evolution of temperature can be reasonably predicted by the proposed model.

1. Introduction

Polymeric materials have been widely used in the manufacture of structural components in the automotive industry, aerospace apparatus, medical devices and civil structures due to their high specific stiffness, moderate strength, transparency and other excellent properties. In practical applications, such structural components are often subjected to a cyclic loading, and the cyclic deformation (including ratchetting) of polymeric materials is very important in the design and reliability assessment of such components.

In the last two decades, the cyclic deformation of polymeric materials was addressed extensively in the experimental and theoretical researches. The experimental observations to the cyclic deformation of polymeric materials including their ratchetting behaviors presented during the asymmetrical stress-controlled cyclic loading were studied by many researchers. These studies demon-strated that the cyclic deformation of polymeric materials is remarkably dependent on the applied stress level (Chen et al., 2015), loading history (Yu et al., 2016), loading rate (Chen et al., 2015), loading path (Wang et al., 2009), ambient temperature (Chen et al., 2016a) and relative humidity (Benaarbia et al., 2014, 2015). Jiang et al. (2013) studied the uniaxial ratchetting of polycarbonate (PC) polymer at room temperature and concluded that the recoverable viscoelastic and irrecoverable viscoplastic strains occur simultaneously during the asymmetric stress-controlled cyclic deformation, and could be separated by the strain recovery test after cyclic deformation. The cyclic deformation of the ultra-high molecular weight polyethylene (UHMWPE) polymer at various loading rates was investigated by Chen et al. (2015). The influences of the relative humidity and loading rate on the cyclic deformation of Polyamide 6.6 (PA6.6) were studied by Benaarbia et al. (2014, 2015). It was shown that the cyclic deformation of the polymeric materials depended on the loading rate strongly (Benaarbia et al., 2014, 2015; Chen et al., 2015). Besides the well-known viscosity of the polymeric materials, the internal heat production plays an important role on the observed rate-dependent cyclic deformation, which can be explained as follows:

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During the cyclic deformation of polymeric materials, the internal heat production comes from the mechanical dissipation and then causes a variation of temperature. The extent of temperature variation depends on the competition of internal heat production against the heat transfer/convection which is basically determined by the loading rate. Higher loading rate results in a more internal heat production and less heat transfer/convection in unit time, and then causes a higher temperature raise. Meanwhile, the material properties of polymers depend on the ambient temperature strongly (Chen et al., 2016a), i.e., the elastic modulus and the yielding point decrease but the viscosity increases with the increasing test temperature. Therefore, the rate-dependent cyclic deformation of polymeric materials is partly determined by the temperature variation due to internal heat production. It means that a thermo-mechanically coupled viscoelastic–viscoplastic constitutive model is urgently necessary to reasonably describe the rate-dependent cyclic deformation of polymeric materials, which considers the internal heat production and the effect of temperature variation on the material properties, simultaneously.

Many constitutive models had been constructed to describe the cyclic deformation of polymeric materials, and can be classified into the following two types:

(1) Viscoelastic models. Schapery (1969, 1997) firstly constructed a one-dimensional nonlinear viscoelastic constitutive model including four nonlinear parameters that are associated with the instantaneous, transient, loading rate and accelerated/decelerated time-dependent responses, respectively. Lai and Bakker (1996) extended the original Schapery's model to a three-dimensional (3D) form and implemented it into a finite element code. After that, the Schapery's model was further modified and implemented into a finite element code by Haj-Ali and Muliana (2004) in order to describe the creep deformation of pultruded and laminated composite materials. The Schapery's model is written in an integral form and its physical background is solid since it is constructed in the framework of irreversible thermodynamics. Xia et al. (2005a, b) developed a viscoelastic constitutive model in a differential form to describe the ratchetting of polymeric materials. A memory surface was introduced into the Xia's model to improve the prediction capability to the shape of stress–strain hysteresis loop. The viscoelastic constitutive model proposed by Xia et al. (2005a, b) in a differential form is relatively simpler than that in an integral form and can be easily implemented into a finite element code. The cyclic deformation of polymeric materials can be well described by the above-mentioned viscoelastic models. However, the viscoplastic deformation occurred during the cyclic deformation of polymeric materials is not considered in these models, the predicted accumulated residual strain predicted by the model at the end of cyclic deformation will be fully recovered after a period of time, which is not consistent with the experimental results.

(2) Viscoplastic models. The models are constructed by extending the viscoplastic ones originally established for the metallic materials. In these models, yielding surface, back stress and isotropic hardening are incorporated and modified. The representative ones can be referred to that developed by Drozdov (2010) and Nguyen et al., (2013). Similar to the viscoelastic models, the cyclic deformation of polymeric materials can be reasonably simulated and predicted by the viscoplastic models. However, the residual strain predicted by the models at the end of cyclic deformation cannot be recovered at all, since the viscoelastic deformation is neglected here, which is also inconsistent with the experimental observations.

Recently, in order to overcome the limitation existed in the above-mentioned viscoelastic and viscoplastic models, some constitutive models (e.g., Miled et al., 2011; Haouala and Doghri, 2015) are developed by considering two inelastic mechanisms, i.e., viscoelasticity and viscoplasticity, simultaneously. Yu et al. (2016) developed a new nonlinear viscoelastic–viscoplastic model to describe the ratchetting of the PC at room temperature. The viscoelastic part in the model proposed by Yu et al. (2016) was obtained by extending the 3D Schapery's model (Lai and Bakker, 1996) in order to consider different nonlinearities of viscoelasticity at various scales of time, and the Ohno–Abdel-Karim's nonlinear kinematic hardening rule was adopted in the viscoplastic part. Both the cyclic stress–strain curve and the time–strain recovery curve can be reasonably predicted by the model of Yu et al. (2016).

It should be noted that the internal heat production and the effect of temperature variation on the material properties of polymeric materials have not been considered in the existing constitutive models. Recently, the thermo-mechanically coupled deformation of polymeric materials has been investigated by Anand et al. (2009) and Srivastava et al. (2010) both experimentally and extensively at monotonic finite strain. More recently, Billon (2012) constructed a physical mechanism based constitutive model by considering the deformation constraint caused by the permanent nodes and slip links. This model was further extended into a thermo-mechanically coupled case by using the irreversible thermodynamics (Maurel-Pantel et al., 2015). The stress–strain curves and the temperature evolutions at large deformation and various loading rates are well described by the models mentioned above. However, the thermo-mechanically coupled cyclic deformation of polymeric materials was not involved there.

Therefore, in this work, based on the assumption of small deformation, a framework of irreversible thermodynamics is first presented to address the contribution of viscoelasticity and viscoplasticity to the thermo-mechanically coupled cyclic deformation of polymeric materials simultaneously. The thermodynamic state of the deformed body is defined by certain state variables, and the Gibbs free energy is then constructed. The internal heat production and the thermodynamic driving forces of some strain-like internal variables are deduced from the dissipation inequality, and the spatiotemporal evolution equation of temperature is obtained by the first law of thermodynamics. Then, the close-formed equations (including the balance equation of stress, evolution equation of temperature, compatibility equation of deformation and constitutive equation) for solving the thermo-mechanically coupled cyclic deformation of polymeric materials are obtained. Based on the experimental observations on the cyclic deformation of the UHMWPE (Chen et al., 2015, 2016a), a specific constitutive model is proposed by adopting some simplifications. To describe the ratchetting of the material more reasonably, a modified kinematic hardening rule is proposed. The proposed model is verified by comparing the predicted results with the corresponding experimental ones of the UHMWPE. It is shown that the temperature- and rate-dependent cyclic stress–strain curves, the evolution of average temperature and the time-strain recovery curve after the cyclic deformation can be reasonably predicted by the proposed model.

2. Constitutive model

2.1. Framework of thermodynamics

The thermodynamic state of the material can be fully defined by certain state variables. The Gibbs free energy G at a material point is decomposed into four parts: i.e., instantaneous elastic energy G^0, viscoelastic energy G^{ve}, viscoplastic energy G^{vp} and the one related to temperature G^T. That is,

$$G = G^0 + G^{ve} + G^{vp} + G^T \tag{1}$$