Mechanics of Materials 103 (2016) 95–109

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Research paper

Locally-exact homogenization of viscoelastic unidirectional composites

Guannan Wang, Marek-Jerzy Pindera*

Civil Engineering Department, University of Viginia, Charlottesville, VA 22904-4742, USA

ARTICLE INFO

Article history:
Received 27 April 2016
Revised 8 September 2016
Available online 10 September 2016

Keywords:
Micromechanics
Homogenization
Periodic microstructures
Elasticity-based solution
Viscoelasticity
Correspondence principle

ABSTRACT

The elasticity-based, locally-exact homogenization theory for periodic materials with hexagonal and tetragonal symmetries is extended to accommodate linearly viscoelastic phases via the correspondence principle. The theory employs Fourier series representations for fiber and matrix displacement fields in the cylindrical coordinate system that satisfy exactly equilibrium equations and continuity conditions in the interior of the unit cell. The inseparable exterior problem requires satisfaction of periodicity conditions efficiently accomplished using previously introduced balanced variational principle which ensures rapid displacement and stress field convergence in the presence of linearly viscoelastic phases with relatively few harmonic terms. The solution's stability and efficiency, with concommitant simplicity of input data construction, facilitate rapid identification of the impact of phase viscoelasticity and array type on homogenized moduli and local fields in wide ranges of fiber volume fraction. We illustrate the theory's utility by investigating the impact of fiber array type and matrix viscoelastic response (constant Poisson's ratio vs constant bulk modulus) on the homogenized response and local stress fields, reporting previously undocumented differences. Specifically, we show that initially small differences between hexagonal and square arrays are magnified substantially by viscoelasticity. New results on the transmission of matrix viscoelastic features to the macroscale are also generated in support of construction of homogenized viscoelastic functions from experimental data.

1. Introduction

The increasing usage of polymeric matrix composites in applications ranging from aerospace, automotive and civil engineering to bioengineering necessitates the development of predictive tools that gauge their long-term behavior. Such knowledge is key to durable and sustainable structural component designs. Polymeric matrix composites exhibit creep and stress relaxation phenomena which need to be understood in order to design durable composite-based components. For instance, time-dependent stress redistribution in a laminated composite plate due to combined stress and relaxation phenomena may lead to local ply-level failure, producing stress transfer leading to local failure at another location, and so on. Characterizing time-dependent response of polymeric matrix composites may be accomplished through experiment. This, however, is time-consuming and costly, and hence typically conducted for a chosen material system with a specific fiber volume fraction.

The alternative to testing is the use of homogenization techniques to characterize the time-dependent response of different fiber/matrix combinations in a wide fiber volume fraction range, validated experimentally against specific material combinations. The simple geometric micromechanics models of unidirectional composites based on a single fiber embedded in the matrix phase, such as the CCA (composite cylinder assemblage), Mori-Tanaka and GSC (generalized self-consistent) models, which may in turn be embedded in the homogenized medium of sought properties, (Christensen, 1979), yield estimates of homogenized moduli but typically do not provide accurate estimates of stress fields that account for adjacent fiber interaction. This may be obtained using numerical or semi-analytical approaches such as the finite-element or finite-volume methods, cf., Pindera et al. (2009) and Charalambakis (2010). These methods provide the means of modeling complex microstructure composites, but demand substantial training on the user's part as well as time-consuming input data construction. Hence interest in elasticity-based homogenization methods for periodic microstructures, see the seminal work of Nemat-Nasser et al. (1982) which has motivated current developments of the eigenstrain expansion technique, has revived within the past 15 years, cf. Wang et al. (2005), Drago and Pindera (2008), Mogilevskaya et al. (2010), Sevostianov et al. (2012), Guinovart-Díaz et al. (2013), Caporale et al. (2015), Wang and Pindera (2015; 2016). The construction of input data for use with these techniques is at least an order of magnitude faster relative

* Corresponding author.
E-mail address: mp3g@virginia.edu (M.-J. Pindera).

http://dx.doi.org/10.1016/j.mechmat.2016.09.009
0167-6636/

to numerical methods, and the execution time is comparable if not faster.

Extensive literature exists that addresses finite-element based, and more recently finite-volume based, homogenization of composite materials containing elastic, elastic-plastic and visco-plastic phases. Substantially fewer contributions are found dealing with viscoelastic response of polymeric matrix composites. The approaches employed for this class of composites include the CCA model, (Hashin et al., 1987); Mori-Tanaka method, (Li et al., 2006); spring models, (Yancey and Pindera, 1990; Jeon and Muliana, 2012); Fourier series-based eigenstrain expansion technique, (Luciano and Barbero, 1995; Caporale et al., 2013); asymptotic homogenization, (Andrianov et al., 2011); and finite-volume technique, (Cavalcante and Marques, 2014). Fewer contributions still may be found that are based on the elasticity approach for periodic composites containing strictly elastic phases within square, hexagonal and tetragonal unit cell architectures, already demonstrated to be an attractive alternative to variational homogenization techniques for this class of problems.

Herein, the elasticity based locally-exact homogenization theory proposed by Drago and Pindera (2008) for rectangular and square periodic microstructures, and Wang and Pindera (2015) for hexagonal arrays with transversely isotropic phases, is further extended to accommodate linearly viscoelastic phase response via the correspondence principle. The theory differs from other elasticity-based solutions of the local unit cell problem such as the eigenstrain expansion technique, (Caporale et al., 2015), the equivalent homogeneity method, (Mogilevskaya et al., 2010), or the eigenfunction expansion technique, (Sevostianov et al., 2012), in the manner of periodic boundary conditions implementation based on a balanced variational principle. This variational principle produces rapid convergence of the displacement field which satisfies exactly the Navier's equations and interfacial continuity conditions in the interior of the unit cell representative of rectangular, square or hexagonal periodic arrays of transversely isotropic inclusions. As a result, converged homogenized moduli and local stress fields alike are obtained with relatively few terms in the displacement field representation. The extended locally-exact homogenization theory that accommodates linearly viscoelastic phases is demonstrated herein to exhibit convergence of both homogenized relaxation moduli (or creep compliances) and local stress fields which is just as rapid.

Section 2 describes the locally-exact homogenization theory's extension which is validated in Section 3. In Section 4 we investigate the combined effects of array type and phase relaxation moduli on the homogenized viscoelastic response and local stress fields, reporting new results, as well as the transmissibility of phase response across scales which is useful in the construction of homogenized response functions from experimental data. Specifically, we address the question whether the homogenized creep compliance elements of a unidirectional composite comprised of a viscoelastic matrix that exhibits power-law creep also exhibit power-law creep response in a wide range of fiber volume fractions. Conclusions are presented in Section 5.

2. Locally-exact homogenization via correspondence principle

We employ the elastic-viscoelastic correspondence principle to transform the solution for the unit cell problem obtained using the locally-exact homogenization theory for periodic composites with transversely isotropic elastic phases to the viscoelastic solution in the Laplace transform domain, cf. Christensen (1971) and Tschoegel et al. (2002). Then we use an efficient inversion method proposed by Zakian (1969), see also Halsted and Brown (1972), to obtain the solution for the homogenized relaxation moduli and creep compliances in the time domain. The alternative approach is to solve the problem in the time domain directly, as for instance done by Cavalcante and Marques (2014) using the generalized FVDAM theory. The advantage of the present approach is that it applies to viscoelastic functions with non-separable and separable kernels alike. The approach, however, depends on the accuracy and efficiency of the chosen Laplace inversion scheme which the Zakian method affords, (Hassanzadeh and Pooladi-Darvish, 2007).

Table 1
Complex coefficients employed in Zakian's inversion formula, Eq. (6), from Laplace transform to time domain.

j	K_j	α_j
1	$-36902.08210 + 196990.4257i$	$12.83767675 + 1.666063445i$
2	$+61277.02524 + 95408.62551i$	$12.22613209 + 5.012718792i$
3	$-28916.56288 + 18169.18531i$	$10.93430308 + 8.409673116i$
4	$+4655.361138 - 1.901528642i$	$8.776434715 + 11.92185389i$
5	$118.7414011 - 141.3036911i$	$5.225453361 + 15.72952905i$

The transformed problem is obtained by replacing the displacement, strain and stress variables in the elastic solution by their Laplace transforms, $u_i \to \hat{u}_i(s)$, $\varepsilon_{ij} \to \hat{\varepsilon}_{ij}(s)$, $\sigma_{ij} \to \hat{\sigma}_{ij}(s)$, and the elastic stiffness matrix elements by their Carson transforms $C_{ijkl} \longrightarrow s\hat{C}_{ijkl}(s)$, where

$$\hat{C}_{ijkl}(s) = \int_0^s C_{ijkl}(t)e^{-st}dt \tag{1}$$

The solution to the unit cell problem in the Laplace-transform domain yields Hill's localization relations, (Hill, 1963), between transformed average strains in the fiber and matrix phases ($k = f, m$) and the transformed homogenized strains in the form

$$\widehat{\bar{\varepsilon}}^{(k)} = \hat{\mathbf{A}}^{(k)}(s\hat{\mathbf{C}}^{(f)}, s\hat{\mathbf{C}}^{(m)}, \upsilon_f)\widehat{\bar{\varepsilon}} \tag{2}$$

which are employed in the construction of the homogenized Hooke's law in the transformed domain. The specified macroscopic strain employed in the determination of the relaxation moduli is $\bar{\varepsilon}(t) = H(t)\bar{\varepsilon}^o$ whose Laplace transform is $\widehat{\bar{\varepsilon}}(s) = \bar{\varepsilon}^o/s$. The homogenized Hooke's law in the transform domain is obtained by averaging local constitutive equations in each phase,

$$\widehat{\bar{\sigma}} = \frac{1}{V}\sum_k \int s\hat{\mathbf{C}}^{(k)}\hat{\varepsilon}^{(k)}dV_k = \sum_k \upsilon_k s\hat{\mathbf{C}}^{(k)}\widehat{\bar{\varepsilon}}^{(k)} \tag{3}$$

where the phase volume fractions obey the relationship $\sum_k \upsilon_k = 1$. Upon use of Eq. (2), the homogenized relationship between stress and strain averages then becomes

$$\widehat{\bar{\sigma}} = \sum_k \upsilon_k s\hat{\mathbf{C}}^{(k)}\hat{\mathbf{A}}^{(k)}\widehat{\bar{\varepsilon}} = s\hat{\mathbf{C}}^*\widehat{\bar{\varepsilon}} \tag{4}$$

where $\hat{\mathbf{C}}^* = \sum_k \upsilon_k \hat{\mathbf{C}}^{(k)}\hat{\mathbf{A}}^{(k)}$. In light of the phase volume fraction relationship above, the homogenized relaxation functions for the unit cell in the Laplace transform domain may be written,

$$\hat{\mathbf{C}}^* = \hat{\mathbf{C}}^{(m)} + \upsilon_f[\hat{\mathbf{C}}^{(f)} - \hat{\mathbf{C}}^{(m)}]\hat{\mathbf{A}}^{(f)} \tag{5}$$

The inversion of the homogenized relaxation functions to the time domain is accomplished by dividing the desired time interval into increments $t = [t_1, t_2, t_3, \ldots, t_N]$ at which the unit cell problem in the Laplace transform domain is solved through the assignment $s(j) = \alpha_j/t_i$ for $j = 1, \ldots, 5$, where the complex values of α_j are given in Table 1. The solution of the unit cell problem at the given time enables calculation of the strain concentration matrix for the fiber phase, $\hat{\mathbf{A}}^{(f)}$, in the above equation. The homogenized relaxation functions at the given time are subsequently calculated according to,

$$\mathbf{C}^*(t_i) = \frac{2}{t_i}\sum_{j=1}^{5} \mathrm{Re}[K_j\hat{\mathbf{C}}^*(\alpha_j/t_i)] \tag{6}$$

where the complex coefficients K_j are included in Table 1. The following section provides an overview of the unit cell solution in the Laplace transform domain that enables calculation of the time-domain relaxation functions based on the above inversion method.