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Dynamic behaviour of a thin laminated plate embedded with auxetic layers subject to in-plane excitation

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ABSTRACT

The dynamic modelling of a simply-supported thin laminated plate subject to in-plane excitation is established based on the classic shear theory and von Kármán nonlinear theory. The method of multiple scales is used to determine an approximate solution for the system. According to solvability conditions, the nonlinear modulation equations arising from the principal parametric resonances are obtained and two possible nontrivial solutions are performed. To analyze the nonlinear dynamic response of the plate embedded with auxetic layers, 5-layered sandwich plate, in which two auxetic elastic layers are alternatively sandwiched between three positive Poisson's ratio (PPR) elastic ones, is presented. The natural frequency of model (m, n) shows an increase with respect to the absolute value of Poisson's ratio. Particularly, the amplitude-frequency responses of the laminated plate subject to principal parametric resonance are analyzed for different values of Poisson's ratio. Moreover, it can be found that for model (m, n), there must be some certain value or interval of negative Poisson's ratio (NPR), which, results in zero response effect, in other words, the in-plane excitation will be ineffective for this model when the Poisson's ratio just lies at such a value or interval. Furthermore, it can also be observed that the certain interval of Poisson's ratio becomes wider with the increase of damping.

1. Introduction

So far, almost all common engineering materials have PPR which is close to 0.3 and less than 0.5, especially aurum has isotropic upper limit whose Poisson's ratio is 0.5. However, these PPR materials have disadvantages such as insufficient shear resistance and vibration absorption so that the applications of man-made materials rise sharply.

Auxetic materials are referred to materials with NPR. They expand laterally in one or more directions when stretched longitudinally, while normal materials contract. They were first described by Love [1] and named by Evans, et al. [2]. Nowadays, auxetic materials are utilized in industrial production as man-made materials, including foams, liquid crystalline polymers and micro/nano-structured polymers [3–6].

With the rapid rise of NPR materials, more and more investigators begin to research the enhancements in the material property for auxetic materials in comparison with the normal materials. For instance, Donescu, et al. predicted the Young's modulus for a laminated periodic material consisted of alternating aluminium and auxetic material by using Bécus homogenization technique [7]. Berinskii found that the re-entrant honeycombs demonstrate auxetic properties for any possible combination of two mechanical parameters: the longitudinal stiffness and the torsional stiffness and regular honeycomb structures can demonstrate both auxetic and non-auxetic properties at different combinations of angle, torsional and longitudinal stiffness [8]. Asemi and Shariyat investigated the effects of auxeticity of the materials on uniaxial and biaxial post-buckling behaviours of the FGM plates by using the exact three-dimensional theory of elasticity, they found that the auxeticity drastically enhances post-buckling behaviour of the FGM plate and more negative Poisson's ratio increases the buckling strength of the plate [9]. Mohanraj, et al. tested the static bending stiffness of the hybrid auxetic composite support both in simulation and experiment, both results are in good agreement with each other, with an error of 6.36% [10]. However, there are some drawbacks which limit the applications of auxetic materials. For instance, the majority of NPR structures only have NPR effects in certain in-plane directions, and the composite structures with isotropic NPR effects are in conceptual stage [11], and because of the substantial porosity of the auxetic materials, their stiffness is high in out-of-plane but low in in-plane [12–14]. Because of the enhancements in mechanical property for auxetic materials in comparison with the normal materials, such as high bending stiffness and high

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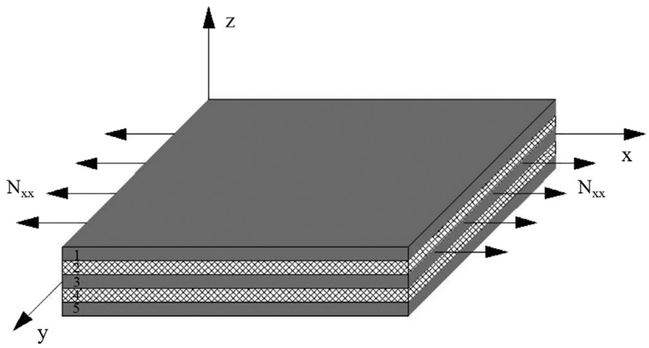

Fig. 1. Topology of a *K*-layered sandwich composite plate.

Young's modulus, auxetic materials are applied in textile industry [15], aerospace and sports [16,17], even biomaterials [18–20] and so on.

Although auxetic materials have had numbers of successful applications up to now, there are few investigations dealt with the analysis of the dynamic mechanism. For instance, Azoti, et al. analyzed a sandwich beam embedded with auxetic layers through a free vibration [21]. Lim investigated the effect of negative Poisson's ratio on the natural frequency of thick plates of arbitrary shape [22]. Boldrin, et al. compared the natural frequency and the modal loss factor of auxetic gradient composite hexagonal honey combs [23]. Then, the complicated nonlinear dynamic behaviours of structures with NPR materials have seldom been investigated up to now.

This paper focuses on the investigation of the complicated nonlinear dynamic behaviour of auxetic composites. Especially, a laminated plate embedded auxetic layers is presented, based on the classic shear theory and von Kármán nonlinear theory, the dynamic equations of the plate subject to in-plane excitation are established. And using multiple scale method, two possible non-trivial solutions are obtained. Moreover, the natural frequencies of the plate for different models are investigated. Particularly, the amplitude-frequency responses of the laminated plate with principal parametric resonance excitation are analyzed for different values of NPR. Furthermore, the supercritical bifurcation and jumping occurring in the laminated plates with specific parameters are revealed.

2. Kinematic formulation

In what follows, a simply-supported thin laminated plate with K layers is taken into account and the classic shear theory is used, as shown in Fig. 1. Thus, we have the displacement as given below,

$$\bar{u}(x,y,t) = u(x,y,t) - zw_{,x}(x,y,t), \tag{1a}$$

$$\bar{v}(x,y,t) = v(x,y,t) - zw_{,y}(x,y,t), \tag{1b}$$

$$\bar{w}(x,y,t) = w(x,y,t), \tag{1c}$$

where $\bar{u}$,$\bar{v}$ and $\bar{w}$ refer to the displacement of any point in the x, y and z direction, respectively. And u, v and w refer to the displacement of intermediate layer, respectively.

According to the elastic theory, we can obtain

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}, \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \chi_{xx} \\ \chi_{yy} \\ \chi_{xy} \end{Bmatrix}, \tag{2}$$

where N and M refer to tensile stress and bending stress, respectively. A_{ij}, D_{ij} are tensile rigidity and bending rigidity, respectively, and expressed as

$$(A_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} (1, z^2) Q_{ij}^k dz, \quad (ij = 11, 12, 22, 66). \tag{3}$$

Suppose that the material for each layer is isotropic, the indentation resistance Q_{ij} for the kth layer is

$$Q_{11}^k = \frac{E^k}{1-(\nu^k)^2}, Q_{12}^k = \frac{\nu^k E^k}{1-(\nu^k)^2}, Q_{66}^k = \frac{E^k}{2(1+\nu^k)}, \quad Q_{22}^k = Q_{11}^k = \frac{E^k}{1-(\nu^k)^2}. \tag{4}$$

where k is the layer number of the plate, E and ν are Young's modulus and Poisson's ratio, respectively.

Suppose that the composite plate is only subject to harmonic in-plane excitations along x direction as shown in Fig. 1, we have the boundary conditions as

$$\int_0^b N_{xx}|_{x=a} dy = \int_0^b -N_1 \cos \Omega_1 \tau \quad dy, \tag{5}$$

where N_1 and Ω_1 are amplitude and frequency of the excitation, respectively.

According to von Kármán nonlinear deformation theory, we finally arrive at

$$A_{11}u_{,xx} + A_{66}u_{,yy} + (A_{12}+A_{66})v_{,xy} + (A_{11}w_{,xx} + A_{66}w_{,yy})w_{,x} + (A_{12}+A_{66})w_{,y}w_{,xy} = 0, \tag{6a}$$

$$A_{22}v_{,yy} + A_{66}v_{,xx} + (A_{12}+A_{66})u_{,xy} + (A_{22}w_{,yy} + A_{66}w_{,xx})w_{,y} + (A_{12}+A_{66})w_{,x}w_{,xy} = 0, \tag{6b}$$

$$D_{11}w_{,xxxx} + D_{22}w_{,yyyy} + 2(D_{12}+2D_{66})w_{,xxyy} - N_{xx}w_{,xx} - 2N_{xy}w_{,xy} - N_{yy}w_{,yy} + \xi w_{,\tau} + \rho h w_{,\tau\tau} = 0. \tag{6c}$$

In what follows, suppose that there are no internal resonances between any two models. Thus, the principal parametric resonance of single model system is taken into consideration due to the existence of damping. So, the bending vibration of the plate is simplified to be a single model system, i.e.,

$$w = W_{mn}(t) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \tag{7}$$

where m and n refer to the vibration modes in x and y direction, respectively.

To achieve both the simply-supported boundary conditions and the balance on both sides of Eq. (6), u and v are supposed to be

$$u = \sin \frac{m\pi x}{a}(a_1 + a_2 \cos \frac{n\pi y}{b} + a_3 \cos \frac{2n\pi y}{b}) + \sin \frac{2m\pi x}{a} \times (a_4 + a_5 \cos \frac{n\pi y}{b} + a_6 \cos \frac{2n\pi y}{b}) + c_1 x, \tag{8a}$$

$$v = \sin \frac{m\pi y}{a}(b_1 + b_2 \cos \frac{n\pi x}{b} + b_3 \cos \frac{2n\pi x}{b}) + \sin \frac{2m\pi y}{a} \times (b_4 + b_5 \cos \frac{n\pi x}{b} + b_6 \cos \frac{2n\pi x}{b}). \tag{8b}$$