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Periodic beam-like structures homogenization by transfer matrix eigen-analysis: A direct approach

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ABSTRACT

The paper deals with a direct approach to homogenize lattice beam-like structures via eigen- and principal vectors of the state transfer matrix. Since the girders unit cells transmit two bending moments, one given by the axial forces, the other originated by nodal moments, the Timoshenko couple-stress beam is employed as substitute continuum. The main advantage of the method is the possibility of operating directly on the sub-partitions of the unit cell stiffness matrix. Closed form solutions for the Pratt and X-braced girders are achieved and used into the homogenization. Unit cells with more complex geometries are numerically addressed with direct approach, showing that the principal vector problem corresponds to the inversion of a well-conditioned matrix. Finally, a validation of the procedure is carried out comparing the predictions of the homogenized models with the outcomes of f.e. analyses performed on a series of girders.

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1. Introduction

Periodic beam-like structures are receiving growing interest from researchers and technicians from several engineering areas. Their optimal trade-off between strength and stiffness, joined with lightness, economy and manufacturing times are the key aspects. Frequent applications in civil and industrial buildings, naval, aerospace, railways and bridge constructions, material design and bio-mechanics can be found [1–14]. Modelling these structures with a 1D homogenized continuum model has great utility in the real problems. While several micropolar models have been reported for the analysis of planar lattices and periodic micro-structures ([15–30], to cite a few), the studies on the micro-polar models for beam-like lattices have not yet achieved the same advances. As far as the authors are aware, only few papers have specifically addressed this topic [31–34].

The present paper introduces a direct approach for the homogenization of periodic beam-like structures by the state transfer matrix eigen-analysis. So far, this methodology has been mostly applied for the dynamic analysis of repetitive or periodic structures ([35–38], i.e.). Only recently, it has also been used for the elasto-static analysis of prismatic beam-like lattices with pin-jointed bars [39–41]. Its practical implementation is problematic since the state transfer matrix **G** is defective and ill-conditioned. Force and displacement transfer methods are presented in [40] to overcome ill-conditioning. By them, a better conditioning is achieved analysing the behaviour of a lattice of n identical cells. Since the proposed method directly operates on the sub-partitions of the unit cell stiffness matrix for searching the unit principal vectors of **G**, all the drawbacks of the transfer methods till now proposed are avoided. Closed form solutions for the unit cell force transmission modes are obtained and used to determine the equivalent stiffnesses of simple girder geometries. The method is easily extended to more complex unit cell geometries too: eigen- and principal vectors of **G** are numerically determined. However, it is shown that the eigen-/principal vector problem is always reduced to the inversion of a well-conditioned 3×3 matrix. The accuracy of the results relative to the homogenized beams in reproducing the behaviour of real discrete beam-like structures is satisfactory. The proposed method is finally assessed with a sensitivity analysis by a set of finite element models.

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2. Direct approach

The unit cells of the analyzed girders are made up of two straight parallel chords rigidly connected to the webs (see Fig. 1). All the cell members are Bernoulli-Euler beams. The top and bottom chords have the same section whose area and second order central moment are named A_c and I_c. The girder transverse webs are assumed axially inextensible to simplify the analysis. This is equivalent to neglect the transverse elongation among the chords during girder deformation. The cross area and the second order moment of the diagonal members are A_d and I_d, while I_w denote the second order moment of the transverse webs. To account the girder periodicity, the two vertical beams of the unit cell will have second order moment equal to the half part of I_w.

To identify any static or kinematical quantity related to the girder *i-th* nodal separating two contiguous cells the sub-script i will be adopted, see Fig. 1d. The superscripts t or b are used to distinguish between the joints or nodes of the same section, depending on whether the top or bottom chord is involved.

Finally, in a coherent manner, top and bottom nodes of the section i are labelled i_t or i_b. In what follows:

$$\boldsymbol{\delta}_i^t = [\, u_i^t \quad v_i^t \quad \varphi_i^t \,]^T \text{ and } \boldsymbol{\delta}_i^b = [\, u_i^b \quad v_i^b \quad \varphi_i^b \,]^T \tag{1}$$

denote the displacement vectors of the joints i^t and i^b, where $u_i^{(\cdot)}$ and $v_i^{(\cdot)}$ are the displacement components of the joint $i^{(\cdot)}$ and $\varphi_i^{(\cdot)}$ is the rotation. Therefore, the displacement vector of the nodal section i is $\boldsymbol{\delta}_i = [\, \boldsymbol{\delta}_i^{t\,T} \quad \boldsymbol{\delta}_i^{b\,T} \,]^T$. Similarly, the nodal forces applied on the cell joints i^t and i^b are:

$$\mathbf{p}_i^t = \left[F_{ix}^t,\ F_{iy}^t,\ m_i^t\right]^T \text{ and } \mathbf{p}_i^b = \left[F_{ix}^b,\ F_{iy}^b,\ m_i^b\right]^T \tag{2}$$

with $F_{ix}^{(\cdot)}$ and $F_{iy}^{(\cdot)}$ respectively the axial and transversal force components and $m_i^{(\cdot)}$ the couple on the joint $i^{(\cdot)}$. Thus, the vector of the nodal forces acting on the section i of the girder is: $\mathbf{p}_i = [\, \mathbf{p}_i^{t\,T} \quad \mathbf{p}_i^{b\,T} \,]^T$. We note that the positive components of $\mathbf{p}_i$ are those acting according the reference axis on the right side of the cell as sketched in Fig. 1d. Thus, the cell i, bounded by the sections $i-1$ and i respectively on the left and right sides will be loaded by the nodal force vectors $-\mathbf{p}_{i-1}$ and $\mathbf{p}_i$.

The unit cells stiffness matrix $\mathbf{K}$ can be computed additively by assembling the stiffnesses of the beam components through the Boolean topological matrices as in the standard finite element analysis.

For our purposes, it is more convenient to adopt static and kinematic alternative quantities to the standard ones of Fig. 1d and Eqs. (1) and (2). More precisely: mean axial displacement $\hat{u}_j = 1/2\left(u_j^t + u_j^b\right)$, section rotation $\psi_j = \left(u_j^b - u_j^t\right)/l_t$ (where l_t is the web length), transverse displacement v and finally the symmetric and anti-symmetric parts of the section nodal rotations $\hat{\varphi}_j = 1/2\left(\varphi_j^t + \varphi_j^b\right)$ *and* $\tilde{\varphi}_j = 1/2\left(\varphi_j^t - \varphi_j^b\right)$ are considered.

The static quantities conjugates of the previous kinematic variables are: the axial force $n_j = (F_j^b + F_j^t)/2$, the bending moment $M_j = \left(F_j^b - F_j^t\right) l_t$ generated by the anti-symmetric axial forces, the shear force $S_j = F_{jy}^t + F_{jy}^b$, the resultant of the nodal moments $\hat{m}_j = m_j^t + m_j^b$ and, finally, the difference between the same moments $\tilde{m}_j = m_j^t - m_j^b$.

The standard kinematic quantities $\boldsymbol{\delta}_j$ can be expressed as functions of the new ones $\mathbf{d}_j = [\, \hat{u}_j \quad \psi_j \quad v_j \quad \hat{\varphi}_j \quad \tilde{\varphi}_j \,]^T$ through the matrix equation $\boldsymbol{\delta}_j = \mathbf{h}\,\mathbf{d}_j$, being:

$$\mathbf{h} = \begin{bmatrix} 1 & -1/2\,l_t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1/2\,l_t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Denoting by $\mathbf{f}_j = [\, \hat{n}_j \quad M_j \quad S_j \quad \hat{m}_j \quad \tilde{m}_j \,]^T$ the vector of the alternative static quantities given by $\mathbf{f}_j = \mathbf{h}^T \mathbf{p}_j$, the unit cell stiffness equation in terms of the variables $\mathbf{d}$ and $\mathbf{f}$ can be written, in a partitioned form, as:

$$\begin{bmatrix} -\mathbf{f}_{i-1} \\ \mathbf{f}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Xi}_{ll} & \boldsymbol{\Xi}_{lr} \\ \boldsymbol{\Xi}_{rl} & \boldsymbol{\Xi}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{d}_{i-1} \\ \mathbf{d}_i \end{bmatrix}, \tag{3}$$

where subscript l and r are used to denote the left and right side of the unit cell and $\boldsymbol{\Xi} = \mathbf{H}^T\mathbf{K}\,\mathbf{H}$ is the cell stiffness matrix, $\mathbf{H}$ being the 10×12 diagonal block matrix having as principal elements the 5×6 $\mathbf{h}$ matrices.

The state vector $\mathbf{s}$ of a nodal cross section of the girder consists of the displacements and forces vectors $\mathbf{d}$ and $\mathbf{f}$. Hence, the state vectors of the end sections of the i cell are $\mathbf{s}_{i-1} = [\, \mathbf{d}_{i-1}{}^T \quad \mathbf{f}_{i-1}{}^T \,]^T$ and $\mathbf{s}_i = [\, \mathbf{d}_i{}^T \quad \mathbf{f}_i{}^T \,]^T$. They are related by the transfer matrix $\mathbf{G}$:

$$\mathbf{G}\,\mathbf{s}_{i-1} = \mathbf{s}_i\,, \tag{4}$$

or equivalently:

$$\begin{bmatrix} \mathbf{G}_{dd} & \mathbf{G}_{df} \\ \mathbf{G}_{fd} & \mathbf{G}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{d}_{i-1} \\ \mathbf{f}_{i-1} \end{bmatrix} = \begin{bmatrix} \mathbf{d}_i \\ \mathbf{f}_i \end{bmatrix} \tag{5}$$

As a consequence, the force transmission modes of the unit cell are given by the unit principal vectors of the $\mathbf{G}$ matrix.

Ill-conditioning of $\mathbf{G}$ can be avoided either solving the eigenvectors problem in closed form or recasting this problem in a numerically non-pathological alternative method. The direct approach pursues this latter strategy. If $\mathbf{s}_e = \left[\mathbf{d}_e^T,\ \mathbf{f}_e^T\right]^T$ is a unit eigen-vector, it is transmitted unchanged through the cell. Furthermore, the principal vector $\mathbf{s}_p$ of the $\mathbf{G}$ matrix, generated by the eigen-vector $\mathbf{s}_e$ is such that $\mathbf{G}\,\mathbf{s}_p = \mathbf{s}_p + \mathbf{s}_e$. Its displacement and force sub vectors $\mathbf{d}_e$ and $\mathbf{f}_e$ are thus linked through the sub-partitions $\boldsymbol{\Xi}_{ij}$ of the stiffness matrix by the equations:

$$\begin{bmatrix} -\mathbf{f}_p \\ \mathbf{f}_p + \mathbf{f}_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Xi}_{ll} & \boldsymbol{\Xi}_{lr} \\ \boldsymbol{\Xi}_{rl} & \boldsymbol{\Xi}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{d}_p \\ \mathbf{d}_p + \mathbf{d}_e \end{bmatrix} \tag{6}$$

These latter relations follow from the stiffness Eq. (5), by substituting the conditions:

$$\mathbf{d}_{i-1} = \mathbf{d}_p, \quad \mathbf{f}_{i-1} = \mathbf{f}_p,$$

$$\mathbf{d}_i = \mathbf{d}_p + \mathbf{d}_e, \quad \mathbf{f}_i = \mathbf{f}_e + \mathbf{f}_p,$$

Adding term by term the two equations in (6), the next condition for the unknown displacement vector $\mathbf{d}_p$ is deducted:

$$\mathbf{f}_e = \mathbf{A}\mathbf{d}_p + \mathbf{B}\mathbf{d}_e \tag{7}$$

with $\mathbf{A} = \boldsymbol{\Xi}_{ll} + \boldsymbol{\Xi}_{lr} + \boldsymbol{\Xi}_{rl} + \boldsymbol{\Xi}_{rr}$ and $\mathbf{B} = \boldsymbol{\Xi}_{lr} + \boldsymbol{\Xi}_{rr}$. By a very similar reasoning, it can be shown that unit eigen-values of $\mathbf{G}$ are such that:

$$\mathbf{A}\mathbf{d}_e = \mathbf{0}. \tag{8}$$

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