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Exact solution for laminar flow in partially collapsed tubes

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Highlights

- Rare, exact solutions for viscous flow in partially collapsed tubes are found.
- These exact solutions describe a wide variety of boundaries, including the important biconcave boundary.
- Examples show the traditional ellipse approximation for a collapsed tube is inappropriate.

Keywords: viscous flow; Poisson equation; closed- form solution; collapsible tube

1. Introduction

The laminar flow in a long tube is basic in fluid mechanics. It is governed by the Poisson equation which is also important in other mechanics problems, such as the torsion of uniform elastic rods. Thus the relevant literature in both fluid and solid mechanics must be consulted.

Closed-form solutions to the Poisson equation has been found for some basic common boundary shapes such as the circle, ellipse, equilateral triangular, annulus, confocal ellipse etc, and infinite series solutions were found for the rectangle, circular sector, annular sector, etc. See e.g. Reviews by Poschl [4], Higgins [2], Wang [7].

We are interested in the fluid flow in deformed circular tubes, especially tubes with biconcave cross sections. Applications include pulmonary flow in the lung, plasma flow in the micro-circulation, and the venous return. These vessels easily become partially collapsed due to interstitial pressure. There exists literature on the shape of collapsed tubes [5, 1, 3, 8]. However, due to the difficulty in the analyses, there are few reports of the flow in such partially collapsed tubes. Only Wang et al [9], using finite elements, computed some flow rates but not the velocity distribution.

This paper seeks an exact, closed-form solution to the flow problem. The method was first