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Short communication

On a damage law for creep rupture of clays with accumulated inelastic deviatoric strain as a damage measure

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ABSTRACT

To avoid the dependency on origin of time, an improved damage law for creep rupture of clays is proposed considering the accumulated inelastic deviatoric strain as a measure of damage, instead of incorporating time directly. This law is incorporated into an existing anisotropic elastoplastic-viscoplastic bounding surface model for clays. The performance of the damage law was demonstrated via the simulations of creep rupture tests on undisturbed clays, and generally a good agreement between model simulations and test data was obtained. Discussions on the creep rupture parameters were followed and further improvement was suggested. At present when high quality test data for creep rupture is very limited, the proposed damage law could serve as a practical way to model creep rupture of clays.

1. Introduction

To describe the behavior of creep rupture of clays, a damage law for viscoplastic response should be incorporated into a constitutive model. Dafalias [1] established a framework for coupled elastopasticity-viscoplasticity for cohesive soils based on the bounding surface plasticity [2] and Perzyna's overstress theory [3]. This framework seems promising in describing the general stress-strain-time relationship and has been used by several researchers, such as Kaliakin [4] and Al-Shamrani and Sture [5]. In the elastoplastic-viscoplastic bounding surface model proposed by Al-Shamrani and Sture [5], a damage law is introduced into the overstress function and some physical explanations are given. As the damage law is explicitly expressed as a function of time, the viscoplastic response is not invariant with respect to the origin of time, which may not be physically reasonable [5,6]. It is probably better to formulate the damage law as a function of some quantities related to time instead of time itself. Kaliakin [4] proposed a simple damage law by considering the accumulated inelastic deviatoric strain as a measure of damage. This law will be improved in this technical note and incorporated into an anisotropic elastoplastic-viscoplastic bounding surface model for clays [7]. Discussions on the improved damage law with accumulated inelastic deviatoric strain as a measure of damage will be made by case studies of creep rupture of undisturbed clays.

2. Model description

The elastoplastic-viscoplastic model [7] is developed by generalizing a previously proposed anisotropic elastoplastic bounding surface model that adopts a flexible shape of bounding surface [8–11], as illustrated in Fig. 1. All the symbols follow those of Jiang et al. [7] and the full model formulations will not be repeated. Here, only some brief explanations related to the viscoplastic response will be given.

In the generalization, Perzyna's overstress theory [3] is adopted thus an overstress reference surface is defined. The inner-most surface is the elastic nucleus; the surface on which the current stress σ_{ij} lies is the loading surface; and the surface in between the two is the overstress reference surface. All the three surfaces are assumed to be nucleuses homologous to the bounding surface with respect to the projection center c_{ij}. Within the overstress reference surface, it is assumed that there is no viscoplastic response. As the stress point moves out of the overstress reference surface, viscoplastic response occurs. The rate of viscoplastic strain is assumed as a function of the "distance" in stress space from the overstress reference surface to the current stress point.

The magnitude of the viscoplastic response is represented by a function Φ called overstress function, and the quantity to measure

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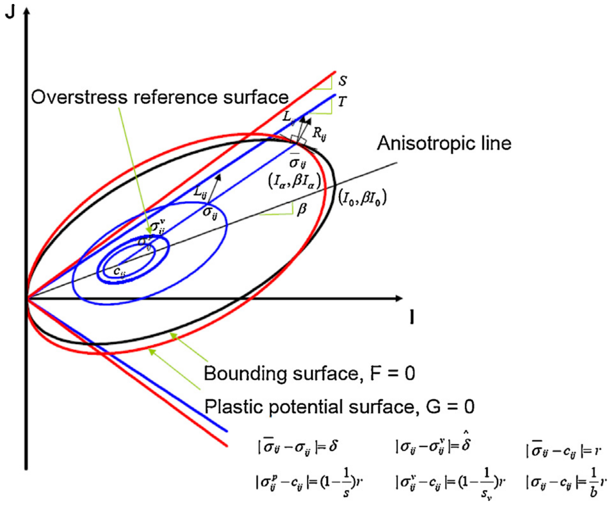

Fig. 1. Elastoplastic–viscoplastic bounding surface model (After Jiang et al. [7]).

the excess stresses beyond the overstress reference surface, $\Delta\hat{\sigma}$, which will be called the "normalized overstress," is defined as

$$\Delta\hat{\sigma} = \frac{\hat{\delta}}{r - \frac{r}{s_v}} \tag{1a}$$

$$\hat{\delta} = |\sigma_{ij} - \sigma^v_{ij}| \tag{1b}$$

where the sign | | represents the norm of a given quantity and σ^v_{ij} is the stress point at the intersection of the mapping line and the boundary of the overstress reference surface. Thus, $\hat{\delta}$ is the "distance" from the overstress reference surface to the current stress point. The overstress function is proposed as

$$\Phi = \frac{1}{V} g_s \left(\Delta\hat{\sigma}\right)^n \tag{2}$$

$$g_s = \exp(\frac{J}{SI}) \tag{3}$$

where V and n are viscoplastic parameters. In the current development, the overstress function is generalized by considering the effect of viscoplastic damage D. One form of the overstress function could simply be

$$\Phi = \frac{1}{V} g_s \left(\Delta\hat{\sigma}\right)^n D \tag{4}$$

A suitable damage law can be incorporated in D. If no damage is considered, then a constant value can be assigned to D, for example $D \equiv 1$, which is the case of Kaliakin and Dafalias [12] and Jiang et al. [7].

3. Damage law and its parameters

3.1. Damage law

The damage function D has been made to be proportional to the magnitude of the viscoplastic response according to Eq. (4). In Kaliakin [4], the proposed damage law is

$$D = f_e \tag{5}$$

where

$$f_e = 1 + < e^i_a - \varepsilon_m > \tag{6a}$$

$$e^i_a = \left(2/3 e^i_{ij} e^i_{ij}\right)^{\frac{1}{2}} \tag{6b}$$

where e^i_{ij} is the deviatoric inelastic strain, with $e^i_{ij} = \varepsilon^i_{ij} - 1/3\varepsilon^i_{kk}\delta_{ij}$ and ε^i_{ij} is inelastic strain and δ_{ij} is the Kronecker delta. The quantity e^i_a is the accumulated inelastic deviatoric strain, and ε_m is the axial strain corresponding to the minimum rate of axial strain during the creep rupture test. The quantity ε_m will be called characteristic strain for convenience in the following discussion. In the expression for f_e, if $e^i_a \le \varepsilon_m$, the Macaulay bracket yields zero and $f_e = 1$, thus the value of the overstress function is not effected. Once $e^i_a > \varepsilon_m$, the expression $f_e > 1$ enters into the overstress function and hence into the viscoplastic strain rate, the result could be a rapid increase in viscoplastic strain.

However, it is noted that e^i_a is only the inelastic part of the total strain while ε_m represents the total strain, hence the two are not physically equal. In undrained triaxial creep tests, numerically ε_m should be equal to the inelastic part e^i_a plus the elastic part e^e_a of the accumulated deviatoric strain in the axial direction. As it is difficult to measure the inelastic part only, adopting the total axial strain ε_m would be more practical. To let the two items in the Macaulay bracket in Eq. (6a) match each other, either ε_m should be replaced by $\varepsilon_m - e^e_a$ or e^i_a should be replaced by the total accumulated deviatoric strain e_a. For convenience, the latter is adopted and a new function g_e is defined. That is,

$$g_e = 1 + <e_a - \varepsilon_m> \tag{7a}$$

$$e_a = \left(2/3 e_{ij} e_{ij}\right)^{\frac{1}{2}} \tag{7b}$$

where e_{ij} is the deviatoric strain.

Although the total accumulated deviatoric strain is adopted in Eq. (7a), physically it is still the accumulated inelastic deviatoric strain that contributes to the potential creep rupture. To better capture the rapid increase in viscoplastic strain, the following damage law is used in the current model:

$$D = (g_e)^\gamma \tag{8}$$

where the parameter γ is used to magnify the effect of damage. For convenience, the parameter γ will be called rupture rate parameter. If $\gamma = 1$ and f_e is used instead of g_e in Eq. (8), the damage law is identical to that of Kaliakin [4]. If $\gamma = 0$, the accumulated inelastic deviatoric strain has no effect on the overstress function, or, it can be said that there is no damage law. If $\gamma > 1$, the damage measured by the accumulated inelastic deviatoric strain will be significant. The reason for adopting the parameter γ to describe the accelerating rate of creep rupture will be better explained in the following case study for an undisturbed alluvial clay.

3.2. Sensitivity analyses of rupture rate parameter through Umeda clay

The proposed damage law, Eq. (8), will be used to analyze the creep rupture of an undisturbed alluvial clay from Japan called Umeda clay, as reported by Sekiguchi [13]. The experimental results were reported by Murayama et al. [14] and the tests have been simulated by Jiang et al. [7]. Each specimen of Umeda clay was isotropically consolidated at 3 kgf/cm^2(294.0 kPa) for 24 h. Then a prescribed deviator stress was applied to the specimen in a single step with the cell pressure kept constant. The deviator stress was maintained and the undrained creep was initiated. Here, only the tests with creep rupture response will be studied, that is, the tests subjected to deviator stresses of 1.99 kgf/cm^2(195.0 kPa) and 2.19 kgf/cm^2(214.6 kPa). The same parameters as in Jiang et al. [7] are adopted:

$$\lambda = 0.343,\ \kappa = 0.105,\ M_c = N_c = 1.47$$

$$G = 128\,\text{kgf/cm}^2(12544\,\text{kPa}), w = 10.0, C = 0.0, s_v = 1.6,$$