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Wave propagation across a finite heterogeneous interphase modeled as an interface with material properties.

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Abstract

Mechanical problems where an interphase exists between two well-defined, and eventually different, materials are of interest. The aim of this paper is to present a simplified model that, for low frequency regime, is appropriate for this situation, that is an interface model with elastic and inertia properties. We present, among the equations of motion, an identification procedure that is valid for any mass density profile along the thickness of the interphase. For evaluating the accuracy of the model, some calculations of the reflection coefficients in some relevant cases are shown. Besides, a finite element method is used as a benchmark for the high frequency regime. It is worth to be noted that the numerical test has been inspired by the problem of the interphase that is formed across the bone-implant boundary.

Keywords: interphase, interface, surface , wave propagation, reflection coefficient.

1. Introduction

This work deals with reflection/transmission problem between two solids which are separated by a layer filled by a heterogeneous material. One potential application concerns the diagnostic of implant stability in bone tissue. By using quantitative ultrasound (QUS) techniques, we may characterize the newly formed bone tissue found in a thin layer surrounding the implant which have been shown to be a determinant factor of implant stability [14]. Two semi-infinite half-spaces are separated by a finite heterogeneous interphase layer. Many attempts have been made to solve this problem by replacing the interphase by an interface with purely elastic properties [2, 3, 6, 17, 16, 11, 20, 19]. The importance of considering inertial properties to the interface has been highlighted in [4, 7, 21]. An elastic wave travels within the first half-space and interacts with the interphase. Reflection and transmission conditions are studied. We have two models that describe the interphase, corresponding to two different approaches. In the first, it is considered a three-dimensional classic functionally dependent (*i.e.*, heterogeneous) non polar elastic body with finite thickness. In the second approach, we model the interphase with a surface (*i.e.* with an interface) with no thickness but with specific surface properties. Such properties will be related to inertia and elastic concepts and characterized, respectively, by kinetic and internal energies of the interface. The kinetic energy, following [21], is assumed to be a quadratic form of the velocities of its boundaries. The internal energy is assumed to be of elastic nature. If the frequency is sufficiently low (*i.e.*, for long wave length), then the assumptions of the second approach are compatible with those of the more sophisticated first one. If the frequency increases, then higher order modes of the interphase are excited and a simple bed of springs is not sufficient to fully consider such a complex situation. We consider, in the numerical case study, the problem of implant-bone interphase, where the properties of the interphase evolves with time due to the remodeling effects [8, 18]. Thus, the necessity to characterize the interphase with parameters that change on a time-scale that is much larger that the period of the incident waves.

2. Finite heterogeneous interphase

2.1. Geometrical configuration

We consider two half-spaces, namely Ω^- and Ω^+, separated by a layer of thickness h, namely Ω^I, as shown in Fig. 1. The domains Ω^- and Ω^+ are modeled as classic homogeneous and isotropic Cauchy continua. The interphase layer Ω^I is modeled as an heterogeneous, isotropic and Cauchy continuum. Let $\mathbf{R}(O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the reference Cartesian frame. The coordinates of a point M in $\mathbf{R}$ are specified by (x_1, x_2, x_3) and the time is denoted by t.

2.2. Equations of motion

In this section, we will write the equations of motion in the different domains. Every unknown is a function of the spatial coordinates and time; every material parameter is constant unless differently specified (*e.g.* in the interphase Ω^I). Superscripts $+$, $-$ or I denote, respectively, the domain Ω^+, Ω^- or Ω^I.

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