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Damping in a parametric pendulum with a view on energy harvesting

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ABSTRACT

The present article addresses the quantification of damping in a parametric pendulum, with a view on further applications in the design of energy harvesting devices. Detailed new experimental data is obtained for such purpose, and a novel mathematical model is presented. Linear and quadratic viscous damping and also dry friction are taken into account. To introduce the dry friction component, the pendulum axis is mounted on ball bearings. This is considered as a very realistic situation of a harvester. Damping parameters are determined by minimizing the difference between numerical and experimental time histories. It is shown that the damping model here presented is more adequate to replicate experiments than commonly used linear models, which consider only a linear viscous damping term characterized by means of free decay tests. It is also pointed that linear models are not adequate for refined studies, since they can lead to erroneous predictions of rotation zones, and consequently to wrong considerations in the design of pendulum harvesters.

1. Introduction

The growing global interest in clean energy has allowed the development of many technologies aimed at energy harvesting from ambient vibrations. This trend has led to review some well-known mechanical systems in the search for suitable harvesting devices. Based on the high kinetic energy available in its rotational motion, the parametric pendulum is one of those systems recently revisited [1–6]. The basic idea of the harvester consists of a pendulum with a vertical motion induced by an ambient energy source. If stable rotations are achieved, a generator attached to the pendulum axis may extract electrical energy. Two sources of ambient vibrations are mainly thought as external excitation: one is the motion of the sea waves, of stochastic nature; and the other is given by the motion of steady vibrating machines, which is generally harmonic and thus easily predictable. This predictability can be used in the design of the pendulum harvester to improve its ability of achieving rotations [7,8].

Damping is an important variable in the design of mechanical harvesters due to its close relation with energy consumption and consequently with efficiency of the system. In fact, it has been pointed out that energy cannot be extracted from a pendulum if damping is very high [2]. It is common practice to assume a damping force due to air drag, which is defined as proportional to the tangential velocity of the bob [2–11]. The proportionality constant of this linear model is usually estimated from a free decay test, assuming an exponential decay of amplitude. This choice is attractive because of its mathematical simplicity, but it is known since a long time that linear damping often cannot represent accurately the behavior of real pendulums [12–14]. This happens mainly because a real pendulum involves sliding or rolling surfaces, such as ball bearings, where Coulomb's nonlinear dry friction cannot be neglected [13]. Besides, if the motion of the pendulum happens at a high Reynolds number, viscous friction may include a quadratic term [13,14] or it may even be purely quadratic [15].

Following those somewhat old but consistent ideas, we propose a mathematical model of the parametric pendulum. To account dry friction, the axis of rotation is assumed to be mounted on ball bearings. We consider this as a realistic situation for an energy harvester, since ball bearings have a good balance between cost, maintenance and friction [16,17]. Besides, linear and quadratic viscous friction terms are also taken into account. The model is derived and tested experimentally considering a reciprocating motion as a parametric excitation. This motion can be found in a wide range of industrial machines, including engines and pumps, where a crank-rod system is used to convert circular motion into linear motion or *vice versa*.

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Nevertheless, the model here presented can be useful for any other excitation of harmonic or stochastic nature. The study is focused on rotational motion which is, as mentioned before, the desired steady state of the pendulum for energy harvesting purposes.

Damping parameters are identified by solving an optimization problem. Identification is based on the minimization of the mean error between experimental data and numerical calculations of the angular position of the pendulum bob. This technique is similar to that used in reference [18], named Fitting Time Histories.

The article is organized as follows. After this introduction (Section 1), we introduce the mathematical model (Section 2) and make a description of the experimental device (Section 3). Then the objective function of the optimization problem is obtained (Section 4). Finally, the results of the study are presented and discussed, focusing on a comparison with a linear damping model (Section 5).

2. Mathematical model

The governing differential equation of the parametrically excited pendulum can be set up by using Lagrange's equation for single-DOF non conservative systems. It is a second-order ordinary differential equation given by

$$ml^2\theta'' + T_v + T_C + ml\left(y'' + g\right)\sin\theta = 0, \tag{1}$$

where m is the mass of the pendulum bob, l the distance between the center of gravity and the axis of the pendulum, g the acceleration of gravity, y the vertical displacement of the axis, θ is the angle measured from the hanging position and T_v and T_C are respectively the viscous friction torque and the Coulomb's friction torque. Derivatives $(\bullet)'$ are performed with respect to time τ.

Let's consider the schematic pendulum-shaker system of Fig. 1. The connecting joint between rod and crank rotates at a constant angular velocity Ω, following a circumferential trajectory. Thus, the displacement of that joint projected horizontally or vertically is exactly sinusoidal in time. Now, the tilt angle of the rod is continuously varying during the cycle of motion. Therefore the linear motion of the upper end of the rod is more complex than a sine function. This linear motion, then transmitted to the pendulum axis, is given by

$$y = r\left(1 - \cos\Omega\tau\right) + L\left(1 - \sqrt{1 - \lambda^2\sin^2\Omega\tau}\right), \tag{2}$$

where r is the crank radius, L is the rod length and $\lambda = r/L$ is the crank/rod ratio.

To determine the viscous damping torque T_v, a drag force has to be established. It has been proposed that this force is neither linear nor quadratic in velocity, but rather a combination of the two [14]. Thus, it seems reasonable to define the drag force as $F_v = aV^2 + bV$, where V is the magnitude of the tangential velocity of the bob and a and b are constant coefficients. Being $V = l\theta'$ and $T_v = l\,F_v$, the viscous damping torque can be expressed as

$$T_v = a\,l^3\left(\theta'\right)^2 sgn\theta' + b\,l^2\theta'. \tag{3}$$

The sign function sgn θ' is needed since both linear and quadratic damping must oppose motion.

The dry friction torque T_C is estimated according to elementary laws of friction, as proportional to the applied force [13,17,19,20]. Hence

$$T_C = \mu\, r_b F_N sgn\theta', \qquad F_N = |ml\left(\theta'\right)^2 + mg\cos\theta|, \tag{4}$$

where μ is the Coulomb friction coefficient, r_b is the bearing bore radius and F_N is the radial dynamic load on the bearings due to the pendulum motion. Since F_N is also the axial load of the pendulum rod, it is calculated by elementary mechanics considerations [21]. Dry friction also opposes motion, thus the sign function $sgn\theta'$ is also present.

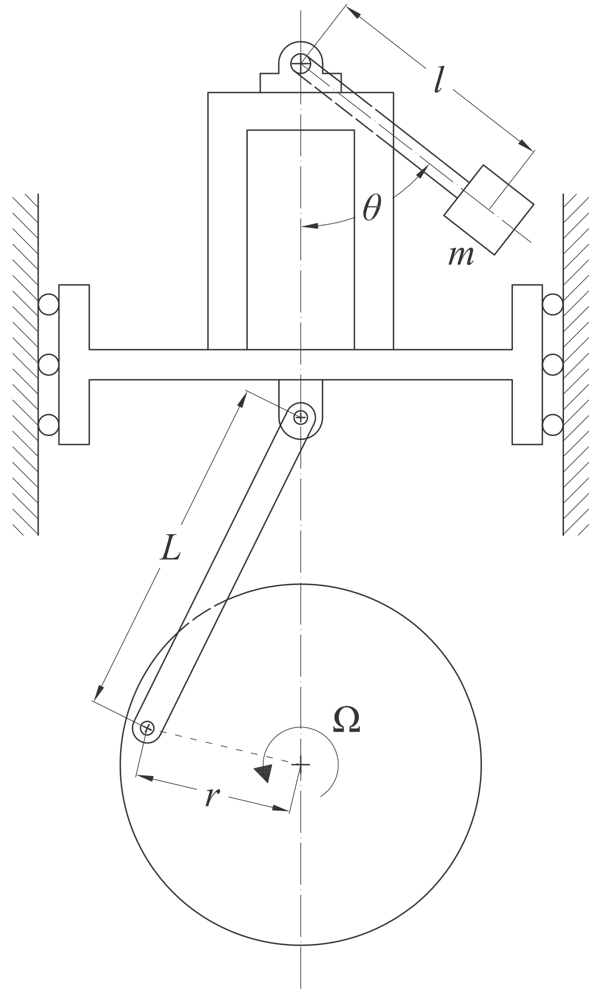

Fig. 1. Schematic pendulum-shaker system, with reciprocating parametric excitation.

Now, introducing Eqs. (2)–(4) into Eq. (1), the non-dimensional equation of motion of the system can be expressed as

$$\ddot\theta + \alpha\,\dot\theta^2 sgn\dot\theta + \beta\,\dot\theta + M|\dot\theta^2 + \cos\theta|sgn\dot\theta + \\ + \left(R\cos\omega t + \lambda^3 R\frac{\Lambda_3}{\Lambda_1^3} + \lambda R\frac{\Lambda_2}{\Lambda_1} + 1\right)\sin\theta = 0, \tag{5}$$

where the following definitions have been made

$$\omega_0 = \sqrt{\frac{g}{l}}, \quad t = \omega_0\tau, \quad \omega = \frac{\Omega}{\omega_0}, \quad R = \frac{r\omega^2}{l} \\ \alpha = \frac{a\,l}{m}, \quad \beta = \frac{b}{m\,\omega_0}, \quad M = \frac{\mu\,r_b}{l}, \\ \Lambda_1 = \sqrt{1 - \lambda^2\sin^2\omega t}, \quad \Lambda_2 = \cos^2\omega t - \sin^2\omega t, \\ \Lambda_3 = \cos^2\omega t\cdot\sin^2\omega t. \tag{6}$$

In Eq. (6), the superimposed dot means the derivative with respect to dimensionless time t. The magnitudes ω, R, α, β and M are non-dimensional parameters associated respectively to the forcing frequency, the forcing amplitude, quadratic viscous damping, linear viscous damping and dry friction. Depending on the settings of these five parameters along with λ, and the choice of initial conditions θ_0 and $\dot\theta_0$, several steady state solutions of Eq. (5) can be obtained, corresponding to different responses of the physical system [7,8]. The most common responses are the rest position, oscillations, rotations and chaotic motion.