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An interface crack with partially electrically conductive crack faces under antiplane mechanical and in-plane electric loadings

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ABSTRACT

An interface crack in a bimaterial piezoelectric space under the action of antiplane mechanical and in-plane electric loadings is analyzed. One zone of the crack faces is electrically conductive while the other part is electrically permeable. All electro-mechanical values are presented using sectionally-analytic vector-functions and a combined Dirichlet-Riemann boundary value problem is formulated. An exact analytical solution of this problem is obtained. Simple analytical expressions for the shear stress, electric field and also for mechanical displacement jump of the crack faces are derived. These values are also presented graphically along the corresponding parts of the material interface. Singular points of the shear stress, electric field and electric displacement jump are found. Their intensity factors are determined as well. Intensity factors variations with respect to the external electric field and different ratios between the electrically conductive and electrically permeable crack face zones are also demonstrated.

1. Introduction

Piezoelectric materials are often used as functional parts of different engineering systems including sensors, transducers and actuators. However, existing micro-defects and particularly interface cracks can strongly reduce their strength. For this reason, interface cracks in piezoelectric materials have been actively studied in the last several decades. Such practically important cases as the in-plane mechanical and electrical loadings and the antiplane mechanical and in-plane electric loadings attracted a considerable attention in scientific literature.

Antiplane problems for electrically permeable and impermeable cracks situated at the interface between piezoelectric layers or between a piezoelectric layer and an elastic layer were considered e.g. in the works by Narita and Shindo [20], Soh et al. [24], Kwon and Lee [10], Li and Tang [12,13], Wang and Sun [26], Feng et al. [5]. Three collinear interface cracks between dissimilar transversely isotropic piezoelectric materials subjected to antiplane mechanical and in-plane electric loadings were analyzed by Choi and Shin [2] and Choi and Chung [3]. The problem of three-layer piezoelectric and elastic strips cracked at the interface was analyzed by Narita and Shindo [19], Kwon and Lee [11]. A moving antiplane crack between two dissimilar piezoelectric solids with account of the Maxwell stress was analyzed by Wang [29].

Moving and circular motionless conductive interface cracks under out-of-plane mechanical loading and in-plane electric loading were studied in papers by Wang et al. [27], and Wang and Zhong [28] and the oscillating singularity at the crack tips was derived. A conductive crack in magnetoelectro-elastic half-space under antiplane mechanical and in-plane electric and magnetic impacts was considered by Rogowski [22]. A more detailed review of antiplane crack problem investigation in piezoelectric bimaterials was presented in the review paper by Govorukha et al. [6].

A crack may arise due to a soft multilayered electrode exfoliation. If an electrode situated at the material interface is completely exfoliated along its whole length, then the formed crack can be considered as a conductive crack. A conductive interface crack for a plane case was considered by Beom and Atluri [1], Loboda et al. [15], Ma et al. [16] for open and contact crack models. Plane fracture problems for interface cracks in piezoelectric and magnetoelectro-elastic bimaterials were considered by Sheveleva et al. [23] and Zhao et al. [31], respectively.

In many cases, the finite length multilayer electrode can completely exfoliate along its whole length together with some part of the material interface, forming a partially electroded (conductive) interface crack. This important problem, to the best of our

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knowledge, has not yet been studied. It constitutes the subject of this study.

2. Basic equations for a piezoelectric bimaterial under antiplane mechanical and in-plane electric loadings

For a piezoelectric material, the following coupled electromechanical equations hold (see e.g. Parton and Kudryavtsev [21])

$$\sigma_{ij} = c_{ijks}\varepsilon_{ks} - e_{sij}E_s,\ D_i = e_{iks}\varepsilon_{ks} + \alpha_{is}E_s, \tag{1}$$

where σ_{ij}, ε_{ij} are the components of stress and strain tensors; D_i, E_i are the components of the electric induction and the electric field, c_{ijks}, e_{sij} are elastic and piezoelectric constants and α_{is} are dielectric constants.

The equilibrium equations in the absence of body forces and free charges are:

$$\sigma_{ij,j} = 0,\ D_{i,j} = 0 \tag{2}$$

The expressions for the deformation and the electric field have the form:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right),\ E_i = -\varphi_{,i}$$

where u_i are the components of the displacement vector and φ is the electric potential.

Assume that the material is transversely isotropic with the poling direction parallel to the x_3-axis. Then for a case of antiplane mechanical and in-plane electric loadings one has

$$u_1 = u_2 = 0,\ u_3 = u_3(x_1, x_2),\ \varphi = \varphi(x_1, x_2).$$

and the constitutive relations (1) take the form:

$$\begin{Bmatrix} \sigma_{3i} \\ D_i \end{Bmatrix} = \boldsymbol{R} \begin{Bmatrix} \partial u_3/\partial x_i \\ \partial \varphi/\partial x_i \end{Bmatrix}, \tag{3}$$

where $i = 1, 2$, $\mathbf{R} = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\alpha_{11} \end{bmatrix}$ and $c_{44} = c_{3232}$, $e_{15} = e_{131}$.

Introducing further the vectors

$\boldsymbol{u} = [u_3, \varphi]^T$ and $\boldsymbol{t} = [\sigma_{32}, D_2]^T$,

the relation (3) for $i = 2$ can be written in the form

$$\boldsymbol{t} = \boldsymbol{R}\frac{\partial \mathbf{u}}{\partial x_2}. \tag{4}$$

Taking into account that the functions u_3 and φ are harmonic, similarly to Suo et al. [25], Zhang and Wang [30], the following presentation for the vector-function $\boldsymbol{u}$ is valid

$$u = 2Re\Phi(z) = \Phi(z) + \bar{\Phi}(\bar{z}), \tag{5}$$

where $\Phi(z) = [\Phi_1(z), \Phi_2(z)]^T$ is an arbitrary analytic vector-function of the complex variable $z = x_1 + ix_2$.Combining (4) and (5) leads to

$$t = \mathbf{Q}\Phi'(z) + \bar{\mathbf{Q}}\bar{\Phi}'(\bar{z}), \tag{6}$$

where $\mathbf{Q} = i\mathbf{R}$.

Let us introduce the following vector-functions

$$\boldsymbol{v}' = \left[u_3', D_2\right]^T,\ \boldsymbol{P} = [\sigma_{32}, -E_1]^T.$$

Taking into account that $u' = \Phi'(z) + \bar{\Phi}'(\bar{z})$ and using the presentation (6), these vector-functions can be written in the form

$$\boldsymbol{v}' = \boldsymbol{A}\Phi'(z) + \bar{\mathbf{A}}\bar{\Phi}'(\bar{z}), \tag{7}$$

$$P = \mathbf{B}\Phi'(z) + \bar{\mathbf{B}}\bar{\Phi}'(\bar{z}), \tag{8}$$

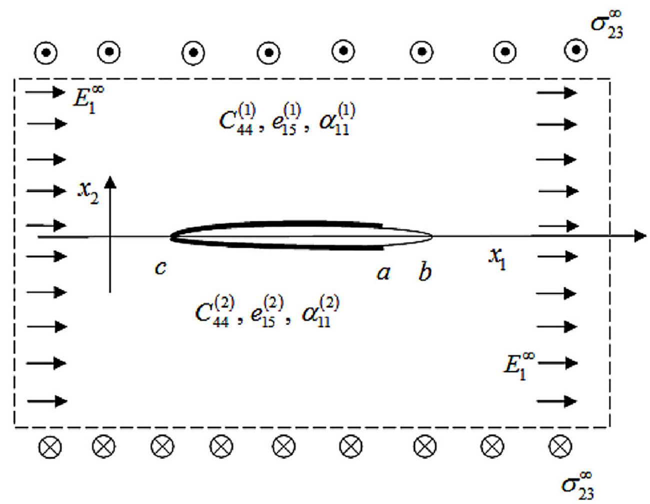

Fig. 1. Interface crack with an electrically conductive (c, a) and an electrically permeable (a, b) zones, $l = b - c$, $\lambda = (b - a)/l$.

where the matrixes **A** and **B** have the following form

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ Q_{21} & Q_{22} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} Q_{11} & Q_{22} \\ 0 & 1 \end{bmatrix}.$$

Assume further that the plane (x_1, x_2) consists of two half-planes $x_2 > 0$ and $x_2 < 0$ having different electromechanical properties. Then using the Eqs. (7) and (8) for each semi-infinite plane and performing analytic continuation procedure, similar to the paper by Suo et al. [25], one gets

$$\langle v'(x_1)\rangle = W^+(x_1) - W^-(x_1) \tag{9}$$

$$P^{(1)}(x_1, 0) = SW^+(x_1) - \bar{S}W^-(x_1), \tag{10}$$

where

$$\langle \mathbf{v}'(x_1)\rangle = \mathbf{v}'^{(1)}(x_1 + i0) - \mathbf{v}'^{(2)}(x_1 - i0) \tag{11}$$

$$S = [\mathbf{A}^{(1)}(B^{(1)})^{-1} - \bar{\mathbf{A}}^{(2)}(\bar{B}^{(2)})^{-1}]^{-1}, \tag{12}$$

$\mathbf{A}^{(m)}$ and $\mathbf{B}^{(m)}$ are the matrices **A** and **B** for upper ($m = 1$) and lower ($m = 2$) regions, respectively; $\langle\cdot\rangle$ means the jump of the function in brackets through the interface. It's worth noting that representations (9) and (10) ensure that equality $\mathbf{P}^{(1)} = \mathbf{P}^{(2)}$ holds true along the whole axis x_1.

For the considered class of piezoelectric materials, we obtain the matrix $\boldsymbol{S}$, which has the following structure

$$\boldsymbol{S} = \begin{bmatrix} is_{11} & s_{12} \\ s_{21} & is_{22} \end{bmatrix}, \tag{13}$$

where all s_{kl} $(k, l = 1, 2)$ are real.

3. Formulation of the problem

Consider a soft double-layered electrode situated at the interval $c \le x_1 \le a$ of the interface $x_2 = 0$ between two semi-infinite spaces $x_2 > 0$ and $x_2 < 0$. Assume that both the electrode itself and the part $a < x_1 < b$ of the interface without electrode are exfoliated. This situation results in an interface crack (c, b), which faces are partially covered with electrodes (Fig. 1). The crack is electrically conductive at $c \le x_1 \le a$ and it is electrically permeable at $a < x_1 < b$ because the absence of the normal crack opening.

An antiplane mechanical and in-plane electric loading is applied at infinity and expressed by the vector $\boldsymbol{P}^{\infty} = \left[\sigma_{23}^{\infty}, E_1^{\infty}\right]^T$. This