Mechanics Research Communications 79 (2017) 24–31

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Chiral effects in piezoelectricity

D. Ieşan

Octav Mayer Institute of Mathematics, Romanian Academy, 700508 Iaşi, Romania

ARTICLE INFO

Article history:
Received 28 August 2016
Accepted 3 December 2016
Available online 21 December 2016

Keywords:
Chiral materials
Piezoelectricity
Concentrated loads
Boussinesq–Somigliana–Galerkin solution

ABSTRACT

This paper is concerned with the linear theory of piezoelectricity for Cosserat continua. The deformation of homogeneous and isotropic chiral materials is investigated. First, a counterpart of the Boussinesq–Somigliana–Galerkin solution in the classical elastostatics is presented. Then, the fundamental solutions in the stationary theory are established. In contrast with the case of achiral solids, in the theory of isotropic chiral materials the displacement and microrotation fields are coupled with the electric field.

1. Introduction

In recent years the behaviour of chiral materials has been the object of intensive research. Chirality can be observed to some carbon nanotubes, bones, honeycomb structures, as well as composites with inclusions. The deformation of chiral elastic materials cannot be described within classical elasticity [1]. The Cosserat theory of elasticity is adequate to describe the deformation of chiral elastic solids (see, e.g., [1–7] and references therein). The Cosserat theory studies continua with oriented particles which have the six degree of freedom of a rigid body [8,9]. This paper is concerned with the theory of chiral piezoelectric solids. The work is motivated by the interest in the study of piezoelectric effect in bones [10,11], carbon nanotubes [12,13], auxetic materials [7] and solids with microstructure (see, e.g., [6,14–16] and references therein). The theory of anisotropic Cosserat elastic bodies subjected to electromagnetic fields was established by Eringen [8,10]. The field equations are obtained for centrosymmetric solids. The theory of isotropic chiral solids has been presented by Lakes [14]. In this paper we consider the theory of piezoelectricity for homogeneous and isotropic chiral Cosserat continua. In Section 2 we present the basic equations. Section 3 presents a counterpart of the Boussinesq–Somigliana–Galerkin solution in the classical elastostatics. In Section 4 we use the representation of solution given in the preceding section to establish the fundamental solutions of the field equations in the stationary theory. The coupling between mechanical and electromagnetic fields is investigated. Section 5 contains representations of Somigliana type for displacements, microrotations and electrostatic potential.

2. Basic equations

We consider a body that in undeformed state t_0 occupies the region B of Euclidean three-dimensional space and is bounded by the piecewise smooth surface ∂B. The deformation of the body is referred to a fixed system of rectangular cartesian axes Ox_k ($k = 1, 2, 3$). We denote by $\boldsymbol{n}$ the outward unit normal of ∂B. We shall employ the usual summation and differentiation conventions.

Throughout this paper we consider the linear theory of homogeneous and isotropic chiral Cosserat piezoelectric solids. Let $\boldsymbol{u}$ be the displacement vector field on B. We denote by $\boldsymbol{\varphi}$ the microrotation vector field. The strain measures are given by (see Eringen [8])

$$e_{ij} = u_{j,i} + \varepsilon_{jik}\varphi_k, \quad \kappa_{ij} = \varphi_{j,i}. \tag{1}$$

where ε_{ijk} is the alternating symbol. Let t_{ij} be the stress tensor and let m_{ij} be the couple stress tensor. The equations of equilibrium can be expressed as

$$t_{ji,j} + f_i = 0, \quad m_{ji,j} + \varepsilon_{ijk}t_{jk} + g_i = 0, \tag{2}$$

E-mail address: iesan@uaic.ro

http://dx.doi.org/10.1016/j.mechrescom.2016.12.002

where $\boldsymbol{f}$ is the body force and $\boldsymbol{g}$ is the body couple. The equations of the electric fields are given by

$$D_{j,j} = q, \quad E_k = -\psi_{,k}, \tag{3}$$

where D_j is the dielectric displacement, q is the volume charge density, E_j is the electric field vector, and ψ is the electrostatic potential.

The constitutive equations for homogeneous and isotropic chiral Cosserat continua can be presented in the form [1,8,16]

$$\begin{aligned} t_{ij} &= \lambda e_{rr}\delta_{ij} + (\mu+\kappa)e_{ij} + \mu e_{ji} + C_1\kappa_{ss}\delta_{ij} + C_2\kappa_{ji} + C_3\kappa_{ij} + \lambda_1\varepsilon_{ijk}E_k,\\ m_{ij} &= \alpha\kappa_{ss}\delta_{ij} + \beta\kappa_{ji} + \gamma\kappa_{ij} + C_1 e_{rr}\delta_{ij} + C_2 e_{ji} + C_3 e_{ij} + \lambda_2\varepsilon_{ijk}E_k,\\ D_k &= -\lambda_1\varepsilon_{ijk}e_{ij} - \lambda_2\varepsilon_{ijk}\kappa_{ij} + \chi E_k, \end{aligned} \tag{4}$$

where δ_{ij} is the Kronecker delta, and λ, μ, κ, α, β, γ, C_1, C_2, C_3, λ_1, λ_2 and χ are constitutive constants. In the case of an achiral material the coefficients C_1, C_2, C_3 and λ_1 are equal to zero. The components of surface traction, the components of the surface moment, and the normal component of the electrical displacement at a regular point of ∂B are given by $t_i = t_{ji}n_j$, $m_i = m_{ji}n_j$, $\sigma = D_k n_k$, respectively. The material stability requires that the strain energy be nonnegative. Thus, the constitutive coefficients must satisfy some restrictions. We note that [8,9]

$$\begin{aligned} &\lambda + 2\mu + \kappa > 0, \quad 2\mu + \kappa > 0, \quad \kappa > 0, \quad \kappa > 0,\\ &\gamma + \beta > 0, \quad \gamma - \beta > 0,\\ &(\lambda + 2\mu + \kappa)(\alpha + \beta + \gamma) - (C_1 + C_2 + C_3)^2 > 0,\\ &\chi > 0, \quad \gamma(\mu + \kappa) - C_3^2 > 0. \end{aligned} \tag{5}$$

3. A solution of the field equations

In this section we establish a counterpart of the Boussinesq–Somigliana–Galerkin solution of the classical elastostatics. It follows from (1)–(4) that the field equations can be expressed in terms of the functions u_i, φ_i and ψ. Thus, in the case of the stationary theory we obtain the following system of equations

$$\begin{aligned} &(\mu+\kappa)\Delta\boldsymbol{u} + (\lambda+\mu)\,\text{grad div}\,\boldsymbol{u} + \kappa\,\text{curl}\,\boldsymbol{\varphi} + C_3\Delta\boldsymbol{\varphi} + (C_1+C_2)\,\text{grad div}\,\boldsymbol{\varphi} = -\boldsymbol{f},\\ &C_3\Delta\boldsymbol{u} + (C_1+C_2)\,\text{grad div}\,\boldsymbol{u} + (\gamma\Delta - 2\kappa)\boldsymbol{\varphi} + (\alpha+\beta)\,\text{grad div}\,\boldsymbol{\varphi} + \kappa\,\text{curl}\,\boldsymbol{u} + 2(C_3 - C_2)\,\text{curl}\,\boldsymbol{\varphi} - 2\lambda_1\,\text{grad}\,\psi = -\boldsymbol{g},\\ &\chi\Delta\psi - 2\lambda_1\,\text{div}\,\boldsymbol{\varphi} = -q, \end{aligned} \tag{6}$$

where Δ is the Laplacian. We note that the displacement and microrotation fields produce an electric field and, conversely, the electric field affects the displacement and microrotation vectors. For achiral materials ($\lambda_1 = 0$) there is no coupling. Let us denote

$$\begin{aligned} &a_1 = (\lambda + 2\mu + \kappa)^{-1}, \quad a_2 = (C_1 + C_2 + C_3)^{-1},\\ &d = (\alpha+\beta+\gamma)(\lambda+2\mu+\kappa) - (C_1+C_2+C_3)^2,\\ &b_1 = (C_1+C_2+C_3)d^{-1}, \quad b_2 = b_1 a_2^{-1},\\ &b_3 = \lambda_1 b_2, \quad d_1 = (\lambda+\kappa)\gamma - C_3^2,\\ &\xi = [2\kappa(\lambda+2\mu+\kappa)d^{-1}]^{1/2},\\ &\nu = [\xi^2 + 4\lambda_1 b_3\chi^{-1}]^{1/2},\\ &\zeta = [\kappa(2\mu+\kappa)d_1^{-1}]^{1/2},\\ &p = 2[(C_3 - C_2)(\mu+\kappa) - \kappa C_3]. \end{aligned} \tag{7}$$

In view of (5) we find that $d > 0$, $d_1 > 0$, $a_1 > 0$, $a_2 > 0$, $\xi > 0$, $\nu > 0$ and $\zeta > 0$. We introduce the operators

$$\begin{aligned} &\Omega = (\mu+\kappa)(\gamma\Delta - 2\kappa) - C_3^2\Delta,\\ &\mathcal{A} = \Omega + \kappa^2 = d_1(\Delta - \zeta^2),\\ &\Sigma_1 = (\lambda+2\mu+\kappa)\Omega - (\mu+\kappa)\mathcal{A},\\ &\Sigma_2 = (C_1+C_2+C_3)\Omega - (C_3\mathcal{A} + p\kappa),\\ &\Sigma_3 = (\alpha+\beta)\Omega\Delta - \kappa^2(\gamma\Delta - 2\kappa) - 2(C_3 - C_2)(p + 2\kappa C_3)\Delta,\\ &S = (\gamma\Delta - 2\kappa)\mathcal{A} + 2(C_3 - C_2)p\Delta, \end{aligned} \tag{8}$$