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The effect of the fractional derivative order on vibrational resonance in a special fractional quintic oscillator

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In this work, we make the general study of the vibratory dynamics of a special fractional quintic oscillator discovered in complex media. First of all, we consider that the system is excited by a combination of both low-frequency force and high-frequency force. Then, we analyse the occurrence of vibrational resonance, where the response consists of a slow motion and a fast motion respectively with low and high frequencies. Through this, we obtain an approximate analytical expression of the response amplitude and we determine the values of the low frequency and the amplitude of the high-frequency force at which vibrational resonance occurs. The theoretical predictions are found to be in good agreement with numerical results. Moreover, for fixed values of the system parameters, varying the order of the fractional derivatives can introduce new vibrational resonance phenomena. We found that low value of the fractional derivative order favor the occurrence of the first vibrational resonance with cross-well motions.

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I. INTRODUCTION

In complex media such as glasses, liquid crystals, polymers, and biopolymers, the dynamical variable of interest often obeys fractional differential equations [1–6].

Fractional differential equations have many applications in applied science and engineering [7–11]. Fractional differential equations have been investigated in pure sciences, such as pure mathematics [12].

In [13], A. Tofighi proposed a new damped fractional oscillator of the form

$$\frac{d^a x}{dt^a} + d\frac{d^b x}{dt^b} + f(x(t)) = 0 \tag{1}$$

where $f(x(t))$ is a linear function of $x(t)$. He showed that the case with $1 < a < 2$ corresponds to attenuated oscillation phenomenon while the case with $2 < a < 3$ corresponds to amplified oscillation phenomenon.

Previous researches on this special oscillator for complex media, were focused on the determination of the unperturbed solution [13–19]. In the present paper, we consider the same system, but disturbed by a biharmonic force. Secondly, we assume that the function $f(x(t))$ is a nonlinear function in order to have a quintic oscillator [20]. Thus, the classical quintic oscillator becomes a special quintic oscillator written as:

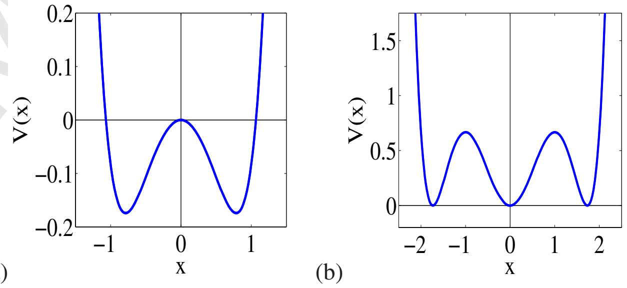
$$\frac{d^a x}{dt^a} + d\frac{d^b x}{dt^b} + \omega_0^2 x + \beta\, x^3 + \gamma\, x^5 = f\cos\omega t + F\cos\Omega t \tag{2}$$

where $1 < a < 2$, $b = a/2$ and $\omega \ll \Omega$ [20–22].

The potential of the system in the absence of damping and external force is

$$V\left(x\right) = \frac{1}{2}\omega_0^2 x^2 + \frac{1}{4}\beta x^4 + \frac{1}{6}\gamma x^6 \tag{3}$$

Depending on the values of the system parameters ω_0^2, β and γ, $V\left(x\right)$ can be a double-well or a triple-well potential [20]. For some values, the different shapes of the system potential are given in Fig.1.



(a) (b)

FIG. 1: (a) Double-well potential of the quintic oscillator for $\omega_0^2 = -1.0$, $\beta = \gamma = 1.0$ and (b) Triple-well potential for $\omega_0^2 = 3.0$, $\beta = -4.0$, $\gamma = 1.0$.

In the system (2), the case where $a = 2$ has been studied in [20]. It has been shown the multiple occurrences of vibrational resonance phenomenon because of the presence of the biharmonic force. The effect of the nonlinear damping on vibrational resonance has been studied in [23]. This nonlinear phenomenon has also been investigated for fractional differential equations. In [24], Yang and Zhu showed the effect of a fractional damping on vibrational resonance while in [25], J.H. Yang et al. studied vibrational resonance in a Duffing system with a fractional generalized delayed feedback. However, it appears that little attention in the literature has been paid to a resonance analysis of the special fractional quintic oscillator, which is why it is the focus of this paper.

The plan of the paper is as follows. In Sec.II, inspired by the theoretical tool in [20, 23], the theoretical analysis for the response amplitude of the output will be carried out for the