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Locally-exact homogenization theory for transversely isotropic unidirectional composites

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Abstract

The locally-exact homogenization theory for unidirectional composites with square periodicity and isotropic phases proposed by Drago and Pindera [18] is extended to architectures with hexagonal symmetry and transversely isotropic phases. The theory employs Fourier series representation for the displacement fields in the fiber and matrix phases in the cylindrical coordinate system that satisfies exactly the equilibrium equations and continuity conditions in the unit cell's interior. The inseparable exterior problem involves satisfaction of periodicity conditions for the hexagonal unit cell geometry demonstrated herein to be readily achievable using the previously introduced balanced variational principle for square geometries. This variational principle plays a key role in the employed unit cell solution, ensuring rapid convergence of the Fourier series coefficients with relatively few harmonic terms, yielding converged homogenized moduli and local stress fields with little computational effort. The solution's stability is illustrated using the dilute case which is shown to reduce to the Eshelby solution regardless of the employed number of harmonic terms. Comparison with published results and predictions of a finite-volume based homogenization in a wide fiber volume range and different fiber/matrix modulus contrast validates the approach's accuracy, and its utility is demonstrated through rapid local stress recovery in a multi-scale application. This extension completes the development of the theory for three important classes of unidirectional reinforcement arrays, thereby providing an efficient alternative to finite-element based homogenization techniques or approximate micromechanical schemes, as well as an efficient standard against which other methods may be compared.

Keywords: micromechanics, homogenization, periodic microstructures, elasticity-based solution

1. Introduction

Most homogenization techniques for periodic materials rely on numerical solutions of the unit cell boundary-value problem using finite-element or finite-volume techniques [1,2]. Interest in elasticity-based methods has revived within the past 15 years in light of advances in computational technology, as well as due to the potential advantages offered by these techniques [3,4]. For instance, microstructural optimization will profit from analytical solutions of unit cell problems due to the significantly smaller design variable space, more efficient specification of objective functions and implementation of more efficient search procedures. Another application is the reconstruction of local fields from homogenized-based results within a multi-scale analysis of local failure modes [5], and material development which relies on rapid answers to what/if questions. Theoretical issues concerning the use of approximations in homogenization schemes may also be addressed by elasticity solutions [6].

Elasticity solutions for the homogenized moduli of periodic heterogeneous materials had been developed by a number of investigators with various degrees of success since the early development stage of composite materials. For unidirectional composites with circular fibers or porosities the main obstacle to an accurate solution is the inseparable nature of the problem due to the different coordinate systems required to solve the interior and exterior unit cell problems. The interior problem involves satisfaction of the elasticity field equations subject to the fiber/matrix continuity conditions most conveniently implemented in the cylindrical coordinate