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MRC-2991; No. of Pages 8
ARTICLE IN PRESS
Mechanics Research Communications xxx (2015) xxx–xxx
Contents lists available at ScienceDirect
Mechanics Research Communications
journal homepage: www.elsevier.com/locate/mechrescom

Multiscale modeling of strain-hardening cementitious composites

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ARTICLE INFO

Article history:
Received 8 February 2015
Received in revised form 8 August 2015
Accepted 10 August 2015
Available online xxx

Keywords:
Strain-hardening
Fracture
Fiber debonding
Lattice modeling
Multiscale characterization

ABSTRACT

This research involves the multiscale characterization of strain-hardening cementitious composites under tensile loading. The sensitivity of cracking behavior to fiber dispersion is studied using a special form of lattice model, in which each fiber is explicitly represented. It is shown that the nonlocal modeling of fiber bridging forces is essential for obtaining realistic patterns of crack development and strain-hardening behavior. Crack count and crack size are simulated for progressively larger levels of tensile strain. The influence of fiber dispersion is clearly evident: regions with significantly fewer fibers act as defects, reducing strength and strain capacity of the material.

1. Introduction

Short dispersed fibers are commonly added to concrete materials to prevent or reduce crack openings. With appropriate constituents and design, such materials strain-harden and exhibit numerous fine cracks, rather than few large ones [1]. These strain-hardening cementitious composites (SHCC) are being developed for many applications, including those that require high toughness or long-term durability. It is understood that crack openings depend on local variations in the fiber content: regions with fewer fibers act as defects within the material and promote larger crack openings [2]. There are various ways to improve composite performance through functional grading of the fiber contents [3]. Multi-scale approaches [4–6] are particularly attractive, as they complement knowledge gained through physical experimentation and elucidate material structure–property relationships. However, approaches that quantitatively relate fiber dispersion and other aspects of material design to cracking behavior are few. To contribute to this need, the authors have developed multiscale models of SHCC, in which individual fibers are explicitly represented within a lattice description of the cement-based matrix [7,8]. The models utilize the micromechanical formulation of Naaman and Namur [9], which connects properties defined local to the fiber–matrix interface and the processes of fiber debonding and pullout after cracking (Fig. 1a, b). The lattice representation of the matrix phase, which is based on the rigid-body-spring concept [10], provides an effective means for upscaling local behavior of the fibers to the composite level (Fig. 1c).

This paper presents new developments in the micromechanical modeling of individual fibers, and their cumulative effects, within fiber reinforced cement composites. To address the limitations of lumped-force (LF) approaches to fiber bridging, we have developed models in which the bridging forces are distributed along the embedded lengths of the fibers [8,11]. Herein, the distributed-force (DF) approach is extended to accommodate nonlinear slip along the matrix–fiber interface during the process of debonding. The consequences of using a DF approach are demonstrated in terms of crack openings, crack spacings, and load–displacement behavior. As expected, both cracking behavior and load–displacement response are correlated to the dispersion of fibers within the material. Comparisons with physical test results demonstrate key capabilities of models based on the DF approach.

2. Lattice model of cement-based matrix

The matrix phase of the tensile specimen considered here is discretized as a series of rigid-body-spring elements, which is a special form of lattice model (Fig. 2). The block-like discretization: (1) facilitates comparisons with theories based on section analysis; (2) enables fine discretization of the longitudinal dimension of the tensile test specimens; and (3) reduces computational effort.

Each lattice element is composed of a zero-size spring set (located at area centroid C of the section common to nodes i and j) and rigid-arm constraints that link the spring set displacements with the nodal degrees of freedom [7,10]. The spring set includes three axial springs (oriented normal and tangential to the section)

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http://dx.doi.org/10.1016/j.mechrescom.2015.08.004

Please cite this article in press as: J. Kang, J.E. Bolander, Multiscale modeling of strain-hardening cementitious composites, Mech. Re. Commun. (2015), http://dx.doi.org/10.1016/j.mechrescom.2015.08.004

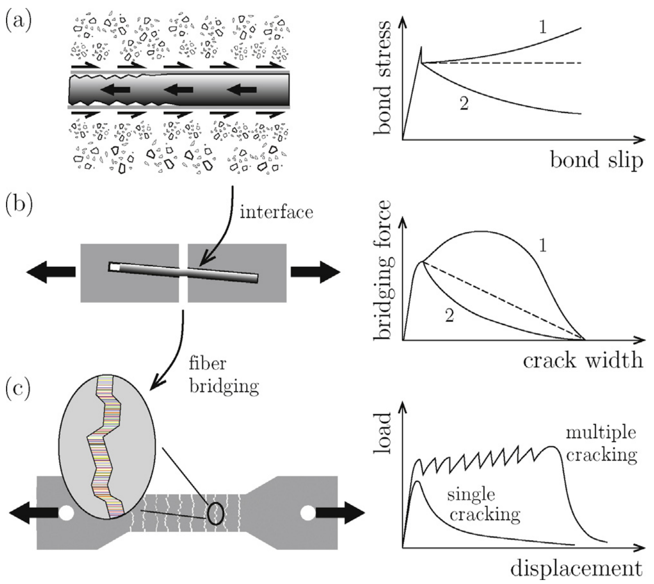

Fig. 1. Multiscale modeling of SHCC in tension: (a) matrix–fiber interface; (b) fiber bridging a crack; and (c) composite behavior.

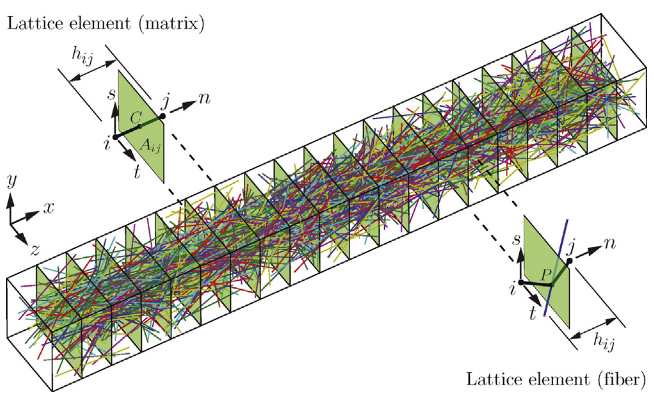

Fig. 2. Lattice discretization of fiber reinforced concrete.

each with stiffness $k = E_m A_{ij}/h_{ij}$, where A_{ij} is the section area, h_{ij} is the element length, and E_m is the matrix elastic modulus. Components of the displacement jump across the zero-size spring set are

$$\delta_x = u_j - u_i + R_{zj}\phi_{yj} - R_{yj}\phi_{zj} - R_{zi}\phi_{yi} + R_{yi}\phi_{zi}$$
$$\delta_y = v_j - v_i - R_{zj}\phi_{xj} + R_{xj}\phi_{zj} + R_{zi}\phi_{xi} - R_{xi}\phi_{zi} \quad (1)$$
$$\delta_z = w_j - w_i + R_{yj}\phi_{xj} - R_{xj}\phi_{yj} - R_{yi}\phi_{xi} + R_{xi}\phi_{yi}$$

where u_j, v_j, and z_j are the x-, y-, and z-displacements of node j, respectively; R_{xj}, R_{yj}, and R_{zj} are the length components of the rigid-arm connecting nodal point j and the spring set location C. Each spring set also includes three rotational springs that are activated by rotations

$$\phi_x = \phi_{xj} - \phi_{xi}$$
$$\phi_y = \phi_{yj} - \phi_{yi} \quad (2)$$
$$\phi_z = \phi_{zj} - \phi_{zi}$$

where ϕ_{xj}, ϕ_{yj}, and ϕ_{zj} are the x-, y-, and z-axis rotations of node j, respectively. Stretching of the axial springs according to Eq. (1) produces corresponding spring forces with magnitudes F_x, F_y, and F_z. A measure of matrix stress is calculated by

$$\sigma_R = \frac{F_R}{A_{ij}^P} \quad (3)$$

where $F_R = (F_x{}^2 + F_y{}^2 + F_z{}^2)^{1/2}$ is the magnitude of the resultant of the spring forces, and A_{ij}^P is the projection of A_{ij} on a plane perpendicular to F_R [12]. For each iteration of the solution process, the ratio $\rho = \sigma_R/f_t$ is calculated for each lattice element, where f_t is tensile strength of the matrix. As is customary for lattice models, only the element with maximum $\rho \geq 1$ experiences fracture within the iteration cycle. The matrix is assumed to be homogenous; material disorder is solely due to the presence of fibers.

3. Fiber inclusions within the lattice model

Fiber placement within the material domain is quasi-random, according to prescribed density functions, as described later in this paper. Alternatively, fiber positions could be found experimentally (e.g., using computed tomography) or from simulations of the casting process, in which the transport of fibers to their final resting positions is modeled [13].

3.1. Pre-cracking representation of fibers

A fiber lattice element is constructed wherever a fiber intersects a matrix element cross-section, as shown in Fig. 2. In a manner comparable to that of the matrix discretization, nodes i and j are linked with rigid-arm constraints to a zero-length spring aligned with the fiber and located at the intersection point P. This spring represents the fiber axial stiffness prior to matrix cracking [7],

$$k_f = \frac{A_f \sigma_f(\xi_P)}{(h_v/\cos\theta)\epsilon_m} \quad (4)$$

where A_f is fiber cross-sectional area; θ is the angle between fiber and loading direction; ϵ_m is the matrix strain in the direction of the fiber; ξ_P is distance from fiber end to point P; and h_v is the distance between nodes i and j in the direction of loading. Elastic shear lag theory [14] is used to determine fiber axial stress at the point of intersection:

$$\sigma_f(\xi_P) = E_f \epsilon_m \left[1 - \frac{\cosh(\beta((l_f/2) - \xi_P))}{\cosh(\beta l_f/2)}\right] \quad (5)$$

in which E_f represents fiber elastic modulus; l_f is the fiber length; and β is a function of several factors, including fiber modulus, radius, arrangement, volume fraction, and the matrix shear modulus at the fiber interface. With k_f thus calculated by Eq. (4), formulation of the element stiffness matrix is analogous to that of the matrix phase.

If N fibers intersect the matrix element cross-section, there will be N fiber element contributions to the stiffness coefficients associated with nodes i and j. Fiber additions do not increase the number of computational degrees of freedom of the model, since fiber elements and matrix elements connect to the same pairs of lattice nodes. This enables computations involving large numbers of fibers. We describe this fiber model as being semi-discrete, because fiber loading is constrained to the rigid-body kinematics of the associated matrix elements. Alternatively, fibers can possess their own degrees of freedom [8], but that capability is computationally expensive even when modest numbers of fibers are considered.

3.2. Post-cracking representation of fibers

3.2.1. Lumped-force (LF) approach

After cracking, properties of the spring component traversing the crack are governed by debonding and frictional pullout of the fiber, according to the micromechanical model of Naaman and Namur [9]. Relevant properties of the fiber–matrix interface include the adhesional and frictional bond strengths, which are represented by τ_a and τ_f, respectively (Fig. 3a). The fiber bridging force is applied