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Research paper

Kinematic analysis and multi-objective optimization of a 3-UPR parallel mechanism for a robotic leg

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In this paper, a parallel mechanism with 3-UPR architecture for a robotic leg application is analyzed for design purposes. The proposed morphology is characterized by the convergence of the three chains to a single point of the moving platform. First, the mechanism is described and its inverse and forward kinematic problems are solved analytically. Its Jacobian matrix is computed to evaluate the singular positions of the end-effector. Then, workspace volume, manipulator dexterity, static efficiency and stiffness are chosen as objective functions for a multi-objective optimization in order to decide the geometrical parameters of the mechanism. The objective functions are mapped in the parameter space and an optimal solution is discussed as suitable for a future prototype.

1. Introduction

Usually, parallel architectures are not used for robotic legs, since they have a smaller workspace than serial ones of the same size, even if they perform better in terms of accuracy and payload [1–3]. A successful example of a 6-degrees-of-freedom robotic leg with parallel structure is presented in the Waseda locomotor in [4]. Several research teams proposed parallel leg structures with limited mobility. Parallel kinematic manipulators with less than six degrees-of-freedom have several advantages over parallel manipulators with six degrees of freedom in terms of simple structure, easy control and lower cost. Usually, translational 3-degrees-of-freedom parallel mechanisms have a larger workspace than 6-degrees-of-freedom ones.

A lot of effort has been done in the last years in order to find an optimal way to analyse and design this kind of structures [5–7]. The work in [5], for example, deals with the type synthesis of lower-mobility parallel manipulators; Zlatanov, Bonev and Gosselin study constraint singularities in [6], which are typical of this class of mechanisms and which cannot be evaluated by the input-output kinematic relation only. Joshi and Tsai present a way to compute a complete 6×6 Jacobian matrix for lower-mobility manipulators in [7]. Some examples of these structures applied in robotic legs are shown in [8–10]. In particular, Wang et al. [8] describe a reconfigurable biped locomotor based on lower-mobility parallel manipulators. Pan and Gao use 3-DoF legs for an hexapod robot [9], while Wang and Ceccarelli [10] proposed a biped robot with 3-UPU legs that has later been developed in the mechanism optimized in this paper [11–13].

The objective functions for the optimization of lower-mobility parallel mechanisms are discussed in many research works. Carbone et al. [14] proposed an optimization procedure for both serial and parallel robots by using workspace volume,

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Jacobian matrix and compliant displacements. Zhang and Gao [15] optimized their manipulator with a novel performance index, the dexterous design, along with reachable workspace evaluated through forward kinematics. Another 3-DoF mechanism is analysed in [16], where the best algorithms for multi-objective optimization are investigated by using the mean value and the standard deviation of the global distributions as the design indices. The work in [17] presents a framework for multi-criteria design optimization of parallel mechanisms with regards to computational efficiency. A method to identify Pareto-optimal solutions for the design of low-mobility parallel manipulators is presented in [18], while both [19, 20] report the optimization of low-mobility parallel manipulators, a linear Delta parallel robot and a symmetric parallel Schönflies-motion generator, respectively.

This paper analyses a novel 3-DoF parallel manipulator that is based on a 3-UPR architecture. The novel tripod structure was introduced in [11–13]. Section 2 presents a kinematic analysis of the manipulator by solving its inverse and direct kinematic problem and by evaluating its Jacobian matrix. Then, in order to find an optimal design for the mechanism, a multi-objective optimization problem is proposed. Among all the indices that are used to evaluate the workspace of the manipulator, its kinematic and dynamic performance and its stiffness, four indices are chosen and computed in their closed-form expressions in Section 3 for the proposed manipulator, while Section 4 solves the multi-objective optimization by mapping the objective functions in the parameter space and discussing the results in order to find an optimal design.

2. Kinematics of the proposed manipulator

The subject of the paper is the 3 degrees-of-freedom mechanism shown in Fig. 1 and introduced in [11–13]. It is composed by a fixed frame and an end-effector body connected to each other by three U$\underline{\text{P}}$R chains, characterized by a universal joint U, an actuated prismatic joint $\underline{\text{P}}$ and a revolute joint R. Referring to Fig. 1(a), each chain consists of a linear actuator with length l_i that is connected to the fixed frame by a universal joint in A_i and to the end-effector by a revolute joint, in H. The mechanism is characterized by a particular arrangement of the universal joints on the base. The joints in A_1 and A_2 are oriented in such a way that the rotation takes place around the Y-axis first, then around the X'-axis. However, the joint in A_3 is oriented so that the rotation order is the opposite - around the X-axis first, and then around the Y' -axis. When this happens, the manipulator behaves like a 3-S$\underline{\text{P}}$R mechanism and can be analysed as such, where S represents a joint with spherical mobility.

Furthermore, the three revolute joints are all located at the end-effector point H thanks to the mechanism shown in Fig. 1(d) and (e): one of the three connected links, labelled link 1 in Fig. 1(d), rotates around the U-axis of the end-effector mechanism, while link 2 and link 3 of Fig. 1(d) can only rotate around the V-axis. This particular configuration of the end-effector simplifies the kinematics of the structure, since the position of point H can be found as the intersection of three spheres centred at A_i with radius equal to l_i, for i = {1, 2, 3}. Therefore, if the base frame is an equilateral triangle with side length a to give

$$ {}^{0}\boldsymbol{A}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix};\ {}^{0}\boldsymbol{A}_2 = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix};\ {}^{0}\boldsymbol{A}_3 = \begin{pmatrix} a/2 \\ \sqrt{3}\ a/2 \\ 0 \end{pmatrix} \tag{1} $$

The inverse kinematic problem of the structure can be solved as

$$ \begin{aligned} l_1 &= \sqrt{x^2 + y^2 + z^2} \\ l_2 &= \sqrt{(x-a)^2 + y^2 + z^2} \\ l_3 &= \sqrt{(x-b)^2 + (y-c)^2 + z^2} \end{aligned} \tag{2} $$

where x, y, z are the absolute coordinates of the end point H.

From Eq. (2) the direct kinematic problem can be expressed as

$$ \begin{aligned} x &= \frac{1}{2a}\left(l_1{}^2 - l_2{}^2 + a^2\right) \\ y &= \frac{1}{2\sqrt{3}a}\left(l_1{}^2 + l_2{}^2 - 2l_3{}^2 + a^2\right) \\ z &= -\sqrt{\frac{-l_1{}^4 - l_2{}^4 - l_3{}^4 - a^4 + l_1{}^2 l_2{}^2 + l_1{}^2 l_3{}^2 + l_2{}^2 l_3{}^2 + a^2\left(l_1{}^2 + l_2{}^2 + l_3{}^2\right)}{3a^2}} \end{aligned} \tag{3} $$

Non-equilateral configuration for the base frame have been investigated but held worse results than the reported ones for the equilateral triangle. This paper analyses and gives the optimization of this particular configuration only.

The Jacobian of the structure can be obtained from differentiation of Eqs. (2) and (3). The result is a 3×3 matrix given by

$$ \boldsymbol{J_p} = \begin{bmatrix} \frac{l_1}{a} & -\frac{l_2}{a} & 0 \\ \frac{l_1}{\sqrt{3}a} & \frac{l_2}{\sqrt{3}a} & -\frac{2l_3}{\sqrt{3}a} \\ \frac{l_1}{z}\left(1 - \frac{x}{a}\right) - \frac{yl_1}{\sqrt{3}az} & \frac{l_2}{az}\left(x - \frac{y}{\sqrt{3}}\right) & \frac{2yl_3}{\sqrt{3}az} \end{bmatrix} \tag{4} $$