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Research paper

# Optimal synthesis of four-bar path generator linkages using Circular Proximity Function

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ABSTRACT

This paper presents a new objective function for the optimization of path-generator four-bar linkages. A four-bar linkage includes four revolute joints, two of which are connected to the coupler link. These two joints, which are known as the moving joints of the linkage, have a remarkable characteristic: both trace circular curves. Using this fact, a new methodology is presented. In this methodology, a dyad is considered which exactly traces the desired path. On the plane attached to the moving link of the dyad, an unlimited number of points can be defined. Among these points, the point which traces a circular curve is very important since this point together with the moving joint of the dyad can be considered as the two moving joints of a four-bar linkage. In order to evaluate the path generated by each point and find the point that traces a near-circular curve, the Circular Proximity Function (CPF) is implemented. Using CPF, a new objective function is introduced that has the lowest number of optimization variables. The optimization process is carried out by the method of differential evolution (DE). Three example problems were solved which resulted in the synthesis of crank-rocker four-bar linkages.

Three example problems are solved in this paper demonstrating the efficacy of the proposed method.

## 1. Introduction

Synthesis of four-bar linkages has been widely studied during the past decades. In general, dimensional synthesis of four-bar linkages is categorized in three classes: function generation, path generation, and motion generation [1]. The problem of path generation synthesis, which is the subject of this paper, is to determine the length of links and the coordinates of the fixed joints in such a way that a specific point on the coupler plane would generate a desired path. The desired path is defined by a set of discrete points known as precision points.

Path generation synthesis of four-bar linkages can be solved analytically [2]. The limited number of precision points is a major drawback of analytical methods. Because of the limited number of independent design variables in a four-bar linkage, analytical synthesis of path-generator four-bar linkages can be done with no more than nine precision points [3].

Optimization methods are alternative solutions which are able solve the problem with more than nine precision points. While in analytical methods the obtained linkage generates a path which exactly falls on the precision points, in optimization methods the obtained linkage approximately generates the desired path and the generated path may not exactly fall on

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the precision points. In this context, a majority of works has been devoted to the optimization of path-generator linkages by means of evolutionary algorithms.

Studies conducted in the field of evolutionary optimization of four-bar linkages can be categorized into two groups. In the first group, researchers focused on the performance of evolutionary algorithms to find better results for this problem. Fang [4], Roston and Sturges [5] were the first researchers who used Genetic Algorithm (GA) to solve this problem. Acharyya and Mandal [6] compared the performance of GA, Particle Swarm Optimization (PSO), and Differential Evolution (DE) for the optimal synthesis of path-generator linkages. They observed that DE shows faster convergence and better results. Laribi et al. [7] combined GA with a fuzzy logic controller to solve this problem. In their method, a fuzzy logic controller monitors the variations of optimization variables in the first run of the algorithm and modifies the boundaries of search space for the second run of GA. Lin [8] combined GA with DE for the optimization of path-generator linkages. Cabrera et al. [9] used DE with a slight modification. They added a mutation operator in order to avoid stagnation and to find the correct optimum linkage. Ortiz et al. [10] also used a modified version of DE in which control parameters of the algorithm are tuned during the execution of the algorithm. Ebrahimi and Payvandy [11] implemented Imperialistic Competitive Algorithm (ICA) for the synthesis of path generating four-bar linkages. Sedano et al. [12] combined evolutionary algorithm with a deterministic optimization technique. The deterministic optimization method used in their study enables the optimization algorithm to avoid the influence of size, location, and orientation of the generated curve. Bulatović et al. [13] studied the application of Krill Herd (KH) algorithm for the dimensional synthesis of path-generator linkages. The paper presented by Lin and Hsiao [14] is one of the latest studies conducted in this framework.

In the second group, researchers concentrated on the performance of objective functions in the optimal synthesis of path-generator linkages. Zhou and Cheung [15] presented a new objective function, which is based on the orientation structural error of the fixed link. Ullah and Kota [16] used Fourier descriptors and presented an effective objective function for the shape optimization of coupler curve. This objective function compares the shape of the generated curve with the desired path without being affected by the size, orientation, or location of the curve. Fernández-Bustos et al. [17] used the finite element method and proposed a new objective function. Matekar and Gogate [18] introduced a modified distance error function and studied its performance in comparison with the Euclidean distance error function. Kim and Yoo [19] employed a spring-connected arbitrarily-sized rectangular block model for the optimization of path-generator linkages. Buśkiewicz [20] presented a new technique for the path generation synthesis of four-bar linkages in that the deviation of generated path of the joints from the ideal path is considered as the error function. Kim et al. [21] used the first and second order derivatives of the coupler curve and proposed a two-step optimization strategy for the path generation synthesis. Each objective function presents a different approach for the optimization of path-generator linkages. The interesting fact about such objective functions is that each takes a different number of optimization variables. For example, the objective function presented in [15] has 6 optimization variables, while the objective function presented in [19] includes 28 optimization variables.

It is necessary to distinguish optimization variables from design variables. The design variables are required to fully define a linkage while the optimization variables are the input parameters of the objective function. The number of design variables for a path-generator four-bar linkage is nine. However, the number of optimization variables depends on our choice of the objective function. For instance, the most common objective function for path generation synthesis known as tracking error (TE) has $9+n$ optimization variables, where $n$ is the number of precision points. This function takes all nine design variables and the angle of the driving link for each generated point as input parameters. The main drawback of using TE as objective function is that when the number of precision points increases, the number of optimization variables also increases. Therefore, high number of precision points results in a large-dimensional search space which significantly increases the computational cost. The objective function presented in this paper has four optimization variables and its number of optimization variables is independent of the number of precision points. Therefore, increasing the number precision points does not change the dimension of the search space.

This paper is closely related to the recent work [22] on the optimal synthesis of four-bar motion generator linkages. In [22], a new objective function was introduced for motion generation synthesis, which reduces the number of optimization variables. In a similar way, a new objective function is presented in this study for path generation synthesis, which has the lowest number of optimization variables in comparison with other objective functions presented so far.

This paper is organized as follows. In Section 2, kinematics of a four-bar linkage and the design variables needed to define a path-generator linkage are presented. Section 3 is devoted to the explanation of the methodology. In Section 4, the optimization algorithm is explained. Performance of the presented method is investigated by solving three example problems in Section 5.

## 2. Kinematics of four-bar path-generator linkages

In Fig. 1, a path-generator linkage with its design variables is depicted. In total, there are nine independent design variables. $r_1, r_2, r_3,$ and $r_4$ determine the lengths of the links, $r_5$ and $\beta$ define the position of the point on coupler plane which generates the path. $x_A, y_A$, and $\alpha$ also define the position and the orientation of the fixed link and therefore the local x,y frame. These nine design variables can fully define a path-generator four-bar linkage.