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Research paper

A benchmark study on accuracy-controlled distance calculation between superellipsoid and superovoid contact geometries

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ABSTRACT

Meshes are considered the gold standard regarding contact geometries of many mechanical models, even those represented with discrete surface contact elements. However, meshes may not be the best formulations when controlled precision and execution time become paramount. In this paper, we address parametric and implicit formulations for precise contact distance estimations between superovoidal shapes, which generalize superellipsoids. Parametric and implicit models provide more compact descriptions than meshes, while making it possible to approximate mechanical parts with great precision. Contrary to meshes, these geometric representations can then support fast calculation of distances with arbitrary precision without paying a storage or computation time penalty. We performed a benchmark study to compare different superellipsoidal and superovoidal contact geometry representations, including implicit surfaces, parametric surfaces and triangular meshes. We tested 10,000 contact pairs and also considered two application cases: robot fingers of an iCub and dental occlusion during bite. Our results show that the implicit model is the most efficient contact geometry representation, followed by parametric and mesh surfaces. In addition, results show that either implicit or parametric superovoids can provide more accurate distance estimations than meshes in practical settings where precise contact points, surface normals and clearance estimations are required.

1. Introduction

One of the most noteworthy interactions between bodies in a mechanical system is collision, i.e., when two bodies physically intersect at one point or region, exerting a force on one another [1,2]. Because collisions greatly influence the behavior of a mechanical system, they must be accurately modeled and accounted for. In Contact Analysis, minimum distance estimation is the operation used to compute whether two objects are intersecting, and if so, where that intersection occurs.

The minimum distance depends on the geometry of the considered objects. Therefore, the geometric representation of object should be chosen wisely, so that the simulated system is accurate enough when compared to its real counterpart. A

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common approach is to approximate the objects by polyhedrons, also called meshes [3]. Since meshes can only represent straight edges and faces, any smooth curves must be discretized. This requires using a large number of vertices to achieve a faithful representation of the surface geometry. These high-resolution meshes lead to slow collision detection algorithms and high memory usage, which hinders their usability for real-time applications. In addition, mesh normals can be very noisy and change abruptly throughout the surface, due to its faceted nature.

Alternatively, objects can be defined using analytical curves and surfaces, such as boxes, spheres, and ellipsoids. In particular, superellipsoids (SE) [4] have been proposed, for which time- and memory-efficient minimum distance algorithms exist [5,6]. Superellipsoids have been used as contact surfaces for robotic haptic feedback devices [7,8], and there are examples of usage of these shapes to represent organic structures with high fidelity, such as human femoral heads [9]. Superellipsoids, together with spheres and ellipsoids, have been extensively studied in many collision detection applications. The superovoids, which generalize the superellipsoidal shape, appear as its natural successors. However, this model has not yet captured the attention of the contact analysis/detection communities. From a geometric modeling point of view, superovoids (SO) while marginally more complex than superellipsoids, can yield a higher accuracy in approximating shapes such as multi-fingered robotic hands, organic structures [9] and everyday life objects. Even the contact geometry of more complex objects with concavities can be approximated by multiple pairs of non-conformal superovoids arranged to fit concave surfaces in conformal contact configurations.

In this paper, we propose a flexible, mathematically well-defined, smooth convex surface called superovoid as a contact geometry model to be used in physical simulations. We present algorithms to compute the minimum distance between different contact geometry representations with (strictly) convex superellipsoidal and superovoidal shapes that interact in a non-conformal fashion. Note that by evaluating the minimum distance between smooth convex objects in a non-conformal configuration, then a quantitative figure reveals the collision state: distances greater than zero reveal that objects are apart; a null distance means that objects touch at a single point; and a “negative” distance represents objects that overlap. The scope of our work covers only non-conformal contact pairs (i.e., convex-convex), where the dimensions of the contact patch are both small compared to the curvatures, and also much smaller than the overall dimensions of the objects. Conformal contact pairs (i.e., convex-concave or concave-concave) are not considered in this paper.

Therefore, the focus of our paper is on the narrow-phase of the contact event and on the minimum distance calculation between (i) very close but still apart surfaces, (ii) single point contact or (iii) surfaces whose overlapping distance is much lesser than the smallest size parameter of a superellipsoid or superovoid. The goal is to determine the influence of contact geometry representation on closest distance calculation between superovoids and superellipsoids in a non-conformal configuration, during the narrow-phase of contact either apart or slightly overlapping. While seemingly minute, precise contact estimations are crucial to evaluating grip force and torques required to hold delicate objects without their breaking or falling to the ground.

We performed benchmark studies covering several aspects related to superovoidal contact geometry representation. In particular, we provide a comparison between implicit, parametric and mesh contact representations along with an extensive battery of tests. Note that comparison with other contact algorithms found in the literature is out of the scope of this paper.

Since meshes can model arbitrary geometries while superovoids cannot, to provide a fair comparison between contact geometries to address the tradeoff between accuracy and computational performance, we considered two case studies with complex objects, namely, robotic grasp motion planning and dental occlusion during chewing or rest, to test non-conformal (i.e., convex-convex) and conformal (i.e., convex-concave or concave-concave) configurations, respectively. As case studies to demonstrate the applicability of the proposed algorithms, we consider contact geometry models of (i) pairs of separated superovoids; (ii) pairs of overlapping superovoids; (iii) iCub robot fingers; and (iv) two molar teeth. We compare the superellipsoid and superovoid approach to an existing mesh-based implementation in terms of accuracy and computation time.

2. Literature review

The literature abounds on work related to collision detection between smooth convex surfaces, in particular, spheres, ellipsoids, and, in a lesser extent, superellipsoids. On the other hand, literature on (super) ovoid collision detection is scarce appearing, at most, in geometric modeling communities [9,10]. Since superovoids are natural extensions of (super) ellipsoids, this section covers recent work on contact detection between ellipsoidal and superellipsoidal shapes, including a couple of papers which served as inspiration to the proposed ovoidal contact algorithms [6,9].

2.1. Contact detection with smooth convex surfaces

Chen et al. 2012 [11] present a method to compute the minimum and maximum distance from a point to an ellipse or ellipsoid. The proposed method involves finding the root of one quartic equation to obtain the closest point to an ellipse, and a sequence of quartic equations for an ellipsoid. An iterative application of the method to find the minimum distance between two ellipsoids is presented, and achieves an error less than 10^{-6} in 11 iterations, but the method was not benchmarked or compared with other algorithms.

Pazouki et al. 2012 [12] propose an efficient and parallelizable method based on ellipsoids for simulating large scale multibody systems, instead of the traditional spheres. The algorithm is divided in three levels. On the lowermost level,