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Compliant multistable tensegrity structures

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ABSTRACT

In this paper, a method to identify compliant tensegrity structures with multiple states of self-equilibrium is described. The considered algorithm is based on the repeated use of a form-finding procedure, using the static Finite-Element-Method. The algorithm can be used in the development process of compliant multistable tensegrity structures with simple topologies, consisting of only few members. As examples four planar tensegrity structures with two or three stable equilibrium configurations are considered and verified experimentally. Furthermore, a specific kind of multistable tensegrity structures, which have identical convex hulls, but differing prestressed states in their equilibrium configurations, is investigated. The potential technical use of the considered structures is discussed in the context of shape-changing locomotion systems using transitions between their equilibrium configurations.

1. Introduction

The consideration of prestressed mechanically compliant structures as mechanisms is a promising recent research direction [12,14,6,3]. A specific class of these structures are compliant free-standing tensegrity structures, consisting of a set of disconnected compressed members connected with a continuous net of compliant tensioned members. Known tensegrity mechanisms use conventional tensegrity structures with only one state of self-equilibrium [1,13,2]. A new specific type of tensegrity structures are multistable tensegrity structures with multiple states of self-equilibrium (equilibrium configurations). In literature only few structures of this kind were previously reported [7,17,11,16,18,5,15].

The consideration of compliant multistable tensegrity structures with members of pronounced elasticity and their use in mechanisms is promising in several aspects:

- Transitions between their equilibrium configurations can be realised with a small control effort, due to their specific movement behaviour (snap-through by force-induced-actuation).
- Due to different geometries and different corresponding prestress states the structures have different mechanical properties (different mechanical compliance) in the different equilibrium configurations. This property can be used to develop effectors with (discrete) tuneable mechanical compliance or (discrete) variable stiffness actuators.
- And additionally, followed by the mechanical properties of conventional compliant tensegrity structures: tuneable mechanical compliance by symmetric prestress adjustment without shape change.

Due to their complex topologies, the use of the known spatial multistable tensegrity structures as mechanisms would be associated with large design/control effort. Therefore, the first task during the realisation of multistable tensegrity

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mechanisms is finding suitable structures with simple topologies, consisting of only few members. A corresponding algorithm is introduced in Section 2, followed by four examples in Section 3, under consideration of planar tensegrity structures with simple topologies. The basic idea for the algorithm is described in [5]. In the present work a more detailed description of the algorithm is given and also additional examples are considered including also two specific kinds of multistable structures. Additionally, selected ideas for the use of mechanisms, based on these structures, in mobile robotics are considered in Section 4. Finally, in Section 5 conclusions and an outlook are given.

2. Theory

The algorithm is based on the repeated use of a form-finding procedure. An overview of form-finding algorithms can be found in [10,8]. The considered form-finding algorithm is based on a geometric nonlinear static Finite-Element-Method without externally applied forces. With the help of the selected form-finding procedure one prestressed equilibrium configuration of a tensegrity structure can be determined (Section 2.1). A systematic variation of input parameters for the form-finding procedure and its repeated use in an external loop is used to detect multiple equilibrium configurations for structures with the considered property (Section 2.2). Due to the applied form-finding method, also the mechanical properties of the structure in the equilibrium configurations (static stability, mechanical compliance) can be determined. Therefore, in contrast to other known methods, the search for structures with given mechanical properties in the equilibrium configurations is conceivable with a further extended version of the considered algorithm. It should be mentioned that this method, and also other known methods (i.e. [17,11,16]), do not guarantee the finding of all possible stable equilibrium configurations for a given structure.

2.1. Form-finding process

The form-finding algorithm is a modified version of the method introduced in [18]. The m members of the considered free-standing tensegrity structures are assumed as massless linear spring elements (parameters: k_j constant longitudinal stiffness and L_{0j}: element free length; $j = 1,2,\ldots m$, see also Appendix A), which are connected by n nodes (pin-joints). At the beginning of the form-finding procedure the elements properties and elements connectivity of the structure are given. The structure is located, with appropriate applied constraint conditions (to exclude all possible rigid-body motions), in an initial configuration, defined by initial declaration of nodal positions. During the form-finding process, the prestress of the structure is carried out by multiple load steps (load increments). The characteristic nonlinear equation

$$\left[K_{\mathrm{red}}\left(\{u_{\mathrm{red}}\}_{(S-1)}\right)\right]\{\Delta u_{\mathrm{red}}\}_{(S)} = \left\{F_{\mathrm{red}}\left(\{u_{\mathrm{red}}\}_{(S-1)}\right)\right\} \quad (1)$$

is solved with an incremental-iterative technique repeatedly (see Fig. 1), to determine the unknown nodal displacements according to the final prestressed equilibrium configuration ($[K_{red}]$: reduced tangent stiffness matrix, $\{\Delta u_{red}\}$: reduced vector of nodal displacement increments, $\{F_{\mathrm{red}}\}$: reduced vector of internal nodal forces, $\{u_{red}\}$: reduced nodal displacement vector). In each iteration (S) $[K_{\mathrm{red}}]$ and $\{F_{\mathrm{red}}\}$ are updated according to $\{u_{red}\}_{(S\text{-}1)}$, then $\{\Delta u_{red}\}_{(S)}$ is calculated, and finally the vector of nodal displacements is updated: $\{u_{red}\}_{(S)} = \{u_{red}\}_{(S\text{-}1)} + \{\Delta u_{red}\}_{(S)}$.

The load is applied in an external loop (see Fig. 1) in N steps:

$$L_{j,g(H)} = (H/N)L_{0j} + ((N-H)/N)L_{j,init} \quad (2)$$

with $H = 1,2,\ldots,N$; $L_{j,g(H)}$: applied element free length of element j in the actual load step H, $L_{j,init}$: length of element j in the initial configuration. According to Eq. (2), during form-finding the element free lengths are adjusted incrementally from an initial value, corresponding to the initial configuration, to the final desired value of L_{0j}. The load incrementation is advisable to enhance the convergence behaviour. The case $N = 1$ would correspond to a pure iterative solution procedure. For each load increment the iteration process is carried out until the global convergence criteria $\mathrm{sum}(|\{\Delta u_i\}_{(S)}|) < \varepsilon_u$ and $\mathrm{sum}(|\{F_i\}_{(S)}|) < \varepsilon_r$ are met, where $\varepsilon_u = 10^{-6}$ mm, $\varepsilon_r = 10^{-6}$ N.

At the end of the form-finding process, in case of convergence for the last load step ($H = N$), static stability of the identified equilibrium configuration can be determined with the help of the eigenvalues of $[K_{red}]$, [19]. The reaction forces related to the applied boundary conditions are zero.

The reduced terms of Eq. (1) are determined from $[K]$ (tangent stiffness matrix), $\{F\}$ (vector of internal nodal forces) and $\{u\}$ (displacement vector) by considering the geometric boundary conditions. The tangent stiffness matrix $[K]$ is the sum of the element stiffness matrices:

$$[K] = \sum_{j=1}^{m} \begin{bmatrix} [\underline{k}_{11}] & [\underline{k}_{12}] & \cdots & [\underline{k}_{1n}] \\ [\underline{k}_{21}] & [\underline{k}_{22}] & & \\ \vdots & & \ddots & \\ [\underline{k}_{n1}] & & & [\underline{k}_{nn}] \end{bmatrix} \quad (3)$$

with $[\underline{k}_{pp}] = [\underline{k}_{zz}] = [\underline{k}_j]$, $[\underline{k}_{pz}] = [\underline{k}_{zp}] = -[\underline{k}_j]$, $[\underline{k}_{rt}] = [I] - [I]$, $[\mathrm{I}] = \mathrm{diag}(1,1,1)$, $[\underline{k}^j] = k_j(\{n_j\}\{n_j\}^T) + P_{jA}/L_{jA}([I] - \{n_j\}\{n_j\}^T)$ [9,10]; $P_{jA} = k_j \bullet (L_{jA} - L_{j,g(H)})$: current internal element force; L_{jA} current element length of element j;