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Research paper

Analytical load sharing and mesh stiffness model for spur/helical and internal/external gears – Towards constant mesh stiffness gear design

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ABSTRACT

In previous works the authors presented two load distribution models for gears. One of the models was based on the rigid tooth assumption and the more complex of the two took advantage of influence coefficients obtained from a FEM model to find the load distribution. In this work an analytical model relying on the ISO 6336 maximum teeth stiffness and a parabolic single tooth stiffness per unit of single line length was developed. This load distribution model relies on an original description of the contact line length based on Heaviside functions to find the gear mesh stiffness. The proposed model is of straight forward implementation, very little computational cost and yields promising results. The concept of constant mesh stiffness gear design is also introduced.

1. Introduction

Gears load distribution has been an object of study for many years now [1–4]. In fact in recent times this has been a very active topic of research. Ajmi [5] developed a model to study the quasi-static and dynamic load distribution of a spur gear. Gear body distortions, time dependent and position properties were considered to simultaneously solve the equations of motion and contact problem. Tooth shape deviations and alignment errors were also taken into account. Pedrero [6] introduced a model of non-uniform load distribution along the line of contact obtained using the minimum elastic potential energy criterion. An approximate, accurate equation for the inverse unitary potential was also suggested. Rincon [7] proposed a model where the deformation at each gear contact point was given by the combination of a local and global term. The global term was calculated using FEM simulations and the local term took advantage of the Hertzian contact theory. More recently Lisle [8] compared the gear root bending stress using the approaches suggested in ISO 6336:2006 and AGMA 2101-D04 with FEM simulations and experimental measurements using strain gauges. Iglesias [9] studied the effect of manufacturing errors in the load sharing in a planetary transmission. Dai [10] investigated the static and dynamic tooth root strains in spur gear pairs using a finite element/contact mechanics approach. Sanchez–Marin [11] proposed a new geometric approach for the tooth contact analysis. Ye [12] proposed an efficient computerized tool for loaded tooth contact analysis that took into consideration the conditions of tip corner contact and shaft misalignment.

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Table 1
Geometrical parameters of the C40, H501 and H951 gears.

Gear type:	C40 $\epsilon_\beta = 0$		H501 $\epsilon_\beta \leq \epsilon_\alpha$		H951 $\epsilon_\beta > \epsilon_\alpha$	
	Driven	Driving	Driven	Driving	Driven	Driving
Number of teeth ($\boldsymbol{z_i}$), [-]	16	24	20	30	38	57
Module ($\boldsymbol{m}$), [mm]	4.5		3.5		1.75	
Centre distance ($\boldsymbol{a}$), [mm]	91.5		91.5			
Pressure angle ($\boldsymbol{\alpha}$), [°]	20		20			
Helix angle ($\boldsymbol{\beta}$), [°]	–		15			
Face width ($\boldsymbol{b}$), [mm]	40		23		23	
Profile shift ($\boldsymbol{x_z}$), [/]	+0.1817	+0.1715	+0.1809	+0.0891	+1.6915	+2.0003
Addendum diameter ($\boldsymbol{d_{ai}}$), [mm]	82.64	118.54	80.67	116.27	76.23	111.73
Transverse contact ratio ($\boldsymbol{\epsilon_\alpha}$), [/]	1.44		1.46		0.93	
Overlap contact ratio ($\boldsymbol{\epsilon_\beta}$), [/]	–		0.54		1.08	
Average roughness ($\boldsymbol{Ra}$), [μm]	≈ 0.7		≈ 0.35		≈ 0.35	
Material	16MnCr5		16MnCr5			

Despite being an age old problem, the amount of work that has been recently done regarding gear load distribution, just shows how important this research topic is. An accurate load distribution profile is fundamental to properly design a gear, not only in terms of load capacity, but also efficiency estimation [13]. The need to develop simpler, more efficient and more accurate load distribution models is then evident, specially considering that the use of optimization algorithms to find optimal designs is becoming a trend [14–16]. Optimization algorithms can be based on iterative processes, therefore having a reliable and fast load distribution model for gears is advantageous if designing optimal gears is a goal.

In a previous work [17] the authors presented two gear load distribution models:

1. Quasi-static rigid model (analytical);
2. Quasi-static local elastic model (numerical-analytical).

The quasi-static rigid analytical model assumed that at a given position in the path of contact the load per unit of length along a line of contact over a tooth was constant. It was also assumed that the load per unit of length was the same between all meshing tooth pairs at a given position, therefore inversely proportional to the sum of the lengths of the lines of contact. This first formulation took advantage of the properties of an approximation of the Heaviside step function to obtain a continuous description of the load distribution based on the lengths of the lines of contact [17]. This model, namely the length of lines of contact model, was reintroduced in this work using an updated notation (Section 2).

The quasi-static local elastic model [17] was based on the constrained minimization of the total potential energy of the gear system. In this model the compliance coefficients were extracted using an open source FEM solver wrapped in a custom code. The load balance including frictional forces was introduced using a Lagrange multiplier [17,18]. As a result of the implementation of this model the load distribution along the lines of contact as well as the gear mesh stiffness are obtained.

In these works [17,18] (and also for the current work) the load distribution problem was studied disregarding dynamic and Hertzian effects. The load dependent Hertzian non-linear effects play a more important role in the mesh stiffness than in the load distribution, where the non-linear effect is diminished [7].

In the current work a quasi-static analytic elastic model was developed and compared with results of previous models. The current model takes the literature definition of stiffness and from there an original approach based on Heaviside functions is combined with the single tooth pair mesh stiffness to obtain a description of the load distribution. In this model, the load distribution along the single line of contact was assumed constant.

The main advantages of the model presented in Section 3 are in its straight forward formulation, simple implementation and accuracy of the load distribution and mesh stiffness results at an insignificant computing cost.

The proposed load distribution model was tested with the different gear geometries presented in Table 1. The C40 and H501 are “conventional” spur and helical gear geometries, while the H951 is a “low loss” gear [19]. It should be noted that the C40 gear is like an FZG type C gear, but with a face width of 40 mm. The planetary gear presented in Table 3 was also used to compare the different models.

2. Quasi-static rigid model

As a first approximation the load per unit of total contact line length along the path of contact was assumed constant. From this assumption a quasi-static rigid load distribution model was developed.

2.1. Analytical description of the length of the lines of contact

Let us consider a coordinate ξ that is the non-dimensional coordinate along the path of contact (distance divided by the transverse base pitch, p_{bt}), which is zero at the starting line of the meshing action (Fig. 1).