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Research paper

Automatic synthesis of planar simple joint mechanisms with up to 19 links

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This paper presents an automatic method to synthesize mechanisms, namely inversions, from planar non-fractionated simple joint kinematic chains. First, the links of a given kinematic chain are classified into several link groups. The mechanisms whose ground links are in distinct link groups are determined to be non-isomorphic. Then, an improved isomorphism identification algorithm, compared to our previous one, is developed to completely eliminate isomorphic mechanisms whose ground links are in the same link group. As a result, a complete set of planar non-fractionated simple joint mechanisms with up to 19 links is synthesized for the first time. The contrastive analysis between our synthesis results and the ones in literature is conducted to demonstrate the validity and efficiency of the synthesis method.

1. Introduction

Structural synthesis of mechanisms is helpful for the innovation and creativity in the design of mechanism-based equipment and system. The traditional approach to identify mechanisms is based on direct visual inspection and intuition. However, using such an approach, it is difficult or impossible to determine feasible, non-isomorphic and complete mechanisms when the design problem becomes too complex. Since the 1960s, numerous relatively efficient approaches have been developed to synthesize various kinds of mechanisms.

Through the use of dual graphs, Sohn et al. [1] developed an automatic procedure to enumerate the kinematic structures of mechanisms. Hwang et al. [2] synthesized the atlas of kinematic chains with two inversions and with up to 15 links. Schmidt et al. [3] presented a general graph grammar methodology for structure synthesis of mechanisms. Li et al. [4] performed the type synthesis of 3R2T 5-DOF (3-rotation 2-translation 5-degree-of-freedom) parallel mechanisms using the Lie group of displacements. Ding et al. [5] synthesized a class of two-layer and two-loop spatial mechanisms with coupling chains based on screw theory. By integrating Assur groups as elements in the group-based adjacency matrix, Li et al. [6] generated a list of 588 types of planar 6-link mechanisms with both revolute and prismatic pairs. He et al. [7] enumerated 38 kinds of 4-DOF parallel press mechanisms based on the GF (generalized function) set theory and then determined the optimal press type. Shao et al. [8] performed the dimensional synthesis of a cam-linkage mechanism based on the kinematic performance criteria and constraints. Yang et al. [9] presented a finite screw approach to synthesize 3-DOF

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translational parallel mechanisms. Recently, Meng et al. [10] analyzed and reviewed several type synthesis approaches of parallel mechanisms, including approaches based on screw theory, the theory of differential geometry, the theory of linear transformation, the POC (position and orientation characteristic) sets and the GF sets.

The synthesis of complete non-isomorphic mechanisms (inversions) from the corresponding planar simple joint kinematic chains has also attracted the attention of researchers for several decades, and substantial investigations dealing with the problem have been implemented. Here, two mechanisms are said to be isomorphic if there exists a one-to-one correspondence between their links (including the ground link) and joints. In other words, two isomorphic mechanisms have the same topological structures and characteristics. In 1971, Quist et al. [11] attempted to enumerate distinct mechanisms from a given kinematic chain and found 1836 10-link 1-DOF mechanisms. In 1984 and 1992, Mruthyunjaya et al. [12,13] enumerated 676 10-link 3-DOF and 7898 11-link 2-DOF distinct mechanisms. In 1991 and 2000, Rao et al. [14,15] developed the hamming number technique to detect isomorphism among mechanisms of a given kinematic chain. In 1994, Chu et al. [16] utilized the link's adjacent-chain table to distinguish mechanisms and confirmed the 1834 10-link 1-DOF mechanisms. In the same year, Vijayananda [17] applied the representation set of links to detect isomorphism and enumerated distinct mechanisms with up to 13 links and seven DOFs. In 1996, Tuttle [18] presented an automatic procedure based on the theory of finite symmetry groups, and generated planar non-fractionated mechanisms with up to 16 links and three DOFs. In 1991, Yan et al. [19] proposed an algorithm, based on combinatorial theory and the concept of permutation groups, to count the number of non-isomorphic mechanisms with the required number and types of links and joints from the candidate kinematic chains. In subsequent papers, Yan et al. [20,21] presented an approach, based on modified permutation groups, generating function and Polya's theory, to identify and count the number of non-isomorphic mechanisms with additional design constraints. In 2009, Simoni et al. [22] enumerated planar non-fractionated mechanisms with up to four independent loops and six DOFs for all screw systems. In 2009, 2013 and 2016, Dargar et al. [23–27] attempted to calculate structural invariants and identification numbers of mechanisms to identify isomorphism. However, their methods were limited to distinguish mechanisms with no more than 10 links. In 2003 and 2011, Mruthyunjaya [28] and Simoni et al. [29] respectively presented a review of the main methods for enumerating mechanisms and summarized the enumeration results.

After reviewing the literature, the study of synthesizing all feasible mechanisms (inversions) without isomorphic structures still has a plenty of room for advances. In the research to date, the synthesis of distinct mechanisms from kinematic chains with relatively complex structures, such as 15 links and four DOFs, has not appeared in the literature. Besides, existing synthesis results derived from different approaches are not completely consistent. The subject has attracted the authors to carry on relevant research, and the purpose of the paper is to present an automatic method to generate a complete list of non-isomorphic mechanisms from planar non-fractionated simple joint kinematic chains, and resolve the contradictions of enumeration results found in the literature.

The entire paper is arranged as follows. In Section 2, several basic concepts including mechanisms of the kinematic chain, single-color and bicolor topological graphs, and fractionated and non-fractionated mechanisms are introduced. In Section 3, the method based on four rules is utilized to classify the links of the given kinematic chain into distinct link groups. In Section 4, an improved isomorphism identification algorithm is developed to completely eliminate isomorphic mechanisms whose ground links are in the same link group. In Section 5, a systematic method is proposed to automatically synthesize non-isomorphic mechanisms from the corresponding simple joint kinematic chains. In Section 6, a complete set of planar non-fractionated mechanisms with up to 19 links is provided, and the discrepancies between our synthesis results and the ones in the literature are discussed exhaustively.

2. Basic concepts

2.1. Mechanisms of the kinematic chain

An *i*-nary joint in a kinematic chain refers to a joint connecting *i* links. If all the joints are binary joints, namely simple joints, the kinematic chain is called a simple joint kinematic chain. If there exists at least one multiple joint ($i > 2$), the kinematic chain is called a multiple joint kinematic chain.

This paper is concentrated on the synthesis of simple joint mechanisms from the corresponding simple joint kinematic chains. Here, a mechanism namely inversion, is defined as a planar kinematic chain with one link fixed (referred to as the ground link). For example, Fig. 1(a) shows a typical 6-link kinematic chain known as the Watt kinematic chain, and Fig. 1(b) and (c) show its two non-isomorphic mechanisms whose ground links are links 1 and 2 respectively. Besides, Fig. 2(a) shows another typical 6-link kinematic chain known as the Stephenson kinematic chain, and Fig. 2(b), (c) and (d) show its three non-isomorphic mechanisms whose ground links are links 1, 2 and 4 respectively.

In order to illustrate the corresponding relationship between the kinematic chain and its topological graph (introduced subsequently), in Fig. 1(a), the six links are labeled as Link 1, Link2 … Link 6, and the seven binary joints are labeled as J1, J2 … J7. For simplicity and clarity, except for Fig. 1(a), the links in other kinematic chains and mechanisms are directly labeled as numbers 1, 2, 3 … etc, and the joints are no longer labeled.

2.2. Single-color and bicolor topological graphs

Since graphs can be used as an aid for the development of computer-aided structural synthesis, the topological graph in graph theory is frequently adopted to represent the topological structures of kinematic chains and mechanisms.